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SPATIAL VARIABILTY OF GROUNDWATER QUALITY IN JAMMU DISTRICT (J&K)

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ABSTRACT

Management of groundwater is very important to meet the increasing demand of water for domestic, agricultural and industrial use. Groundwater quality deterioration is receiving increased attention, especially in densely populated regions. This has led to considerable interest in the design of investigative studies and monitoring programs to describe groundwater quality over various sizes of areas. Various management measures need to know the spatial and temporal behaviour of groundwater.

The spatial dependence between observations can be expressed by the semivariance which is a measure of the average similarity between observations at a given distance apart. One of the prime reasons for obtaining a semivariogram is to use it for estimation of the variable at unsampled locations..

In this report, an application of kriging technique is shown to quantify the spatial variability and to interpolate the groundwater quality as measured in part of Jammu District of J&K State. Experimental semivariogram were computed for nine groundwater quality variables and the theoretical models were fitted to each of these. Various groundwater quality variables (i.e. pH, potassium, magnesium, conductance, sodium, and chloride) were found to be spatially autocorrelated, whereas, variables such as alkalinity, calcium and hardness were found to be non-autocorrelated at the sampling scale. pH was found to be spatially autocorrelated upto a distance of 10km, magnesium, chloride and potassium upto a distance of 14km and conductance upto a distance of 34km. The semivariogram model obtained from the semivariance analysis was used to estimate observations in the unsampled locations within the area. The so obtained values were used for groundwater quality contour mapping.

1.0 INTRODUCTION

All over the world, groundwater is one of the major sources of water. However, the dynamics of groundwater is affected by a number of natural and anthropogenic factors. As groundwater use has increased, issues associated with the quality of groundwater resources have likewise grown in importance. For many years, attention has been directed at contamination from point sources. More recently, concerns have increased about nonpoint sources of contaminent and about the overall quality of groundwater resources.

Groundwater contamination can be detected by analyzing well water for a series of dissolved ion species. Hence, water quality data sets typically contain many variables measured at several spatially scattered locations. As the variables are generally correlated, it is natural to presume that they reflect some common underlying factors. The geological characteristics of the aquifer or the type of landuse are possible factors that govern variations in groundwater chemical composition (Goovaerts et al, 1993). Some of these factors are likely to have a short range action whereas others operate at large spatial scales. As a result, variables must be expected to correlate in a way that is scale dependent. By studying the scale dependent correlation structure of the variables, one can thus hope better to describe these postulated underlying factors (Goovaerts et al, 1993).

Appropriate groundwater quality management measures need reliable quantitative information on groundwater quality behaviour. There exists a need to study the spatial behaviour of groundwater quality. Knowledge of spatial variability of groundwater quality is essential for making reliable ground water quality interpretations and for making accurate predictions of quality at any particular location in the aquifer.

Also, with the advent of high speed computers, which can handle large volume of data easily, more and more distributed models are becoming available. Most of these groundwater models require the input to be available on a grid pattern. But in the field, these parameters are generally measured at random points. So, interpolation of parameters at the grid nodes is a prerequisite to the use of this data in groundwater modelling. Results of these modelling attempts are only as good as the input information. According to Mulla (1988), a major ongoing challenge to watershed modelling is to treat the spatial variation in soil factors more quantitatively, and reduce the extent of spatial averaging that is implicit in the present models. According to Rehfeldt et al. (1992), a means of quantifying the spatial variability, for example of aquifer hydraulic conductivity which controls the movement and dispersion of groundwater solutes, at a reasonable expense is essential for the application of solute transport models to practical problems. However, these measurements are generally carried out randomly in the field. Most of the groundwater models require these measurement at a prespecified grid. So, there is always a need to interpolate the measured parameters at the grid points.

The average distance over which the spatial variation in the groundwater quality parameters are correlated may be called the correlation scale. This scale plays an essential role in problems which require estimating spatial variations of a given water quality parameter from limited observation points and in the solution of transport problems in heterogeneous media using stochastic modelling. The correction scale of a given water quality parameter may be estimated from the correlogram or the semivariogram of that property which, in turn, may be estimated from the available data.

The superior performance of geostatistical methods over other methods is shown by Laslett et al (1987) and Laslett & McBratney (1990) for spatial predicting of soil pH. In this report, Geostatistics (Journel and Huijbregts, 1978) is applied to study the spatial variability of ground water quality in the part of Jammu district of J&K State, India. The interpolation of groundwater quality parameters has been also carried out.

2.0 LITERATURE REVIEW

The theory of Regionalized variables (geostatistics) has received considerable attention in different fields of science in recent years and has been applied to quantifying the spatial variability of different properties.

Semivariograms have been used for spatial modeling of the regional groundwater data by Myers et al (1982). The data used covered more than 10 elements in groundwater from two geologic regions in the Texas (USA). Variograms were found useful for quantifying the differences in spatial variability for elements within a geologic unit and for elements in different geologic units. Geostatistical technique was used to study the spatial variability of groundwater depth data by Dahiya et al (1986). It was concluded that contour maps could serve as a better background for making appropriate decisions in the management of groundwater in the area. Yates et al (1986) have studied the spatial variability of virus inactivation rates in groundwater and found that virus inactivation rates were spatially correlated. Seyhan et al (1987) have applied geostatistical techniques for spatial analysis of hydrochemical variables and groundwater levels in two study areas, namely Sasso Lungo reef stock (northern Italy) and Luz de Tavira area (Southern Portugal). Yates and Yates (1987) have used geostatistical method for estimating virus inactivation rates in groundwater. Rouhani and Hall (1988) have used geostatistics for the design of a groundwater quality monitoring network for a shallow aquifer in the Dougherty Plain, Georgia. Candela et al (1988) have used kriging to judge the sampling efficiency of a ground water quality control observation network in the Llobregat delta deep aquifer.

Geostatistical methodology has been applied to groundwater contaminants at the Chem-Dyne toxic waste site located in Hamilton, Ohio by Cooper and Istok (1988a,b). Six contaminants (~~boron,~~ barium, iron, manganese, zinc and total volatile organic compounds) measured at 36 locations were analyzed. It was concluded that this methodology can be applied for mapping of contaminant distribution in groundwater to obtain reasonably good results. In their third paper (Istok and Cooper, 1988), they have given theoretical and numerical procedures to obtain global estimates and global estimation errors for the mean contaminant concentration using the local estimates.

Samper and Newman (1989) have applied geostatistics to eleven hydrochemical variables and two isotopes from the Madrid basin in Spain. Bjerg and Christensen (1992) have used geostatistics to study the spatial variation in the groundwater quality of a shallow sandy aquifer in the western part of Denmark. Variogram analysis showed that groundwater pH was correlated upto distance of 10m, alkalinity, calcium and potassium were autocorrelated upto a distance of about 5 to 8 m whereas nitrate and chloride showed pure nugget effect. Various other authors (Smyth and Istok, 1989; Spruill and Candela, 1990) have used geostatistics in ground water quality applications.

Interdependency among observations of soil physical properties has been studied by means of autocorrelation analysis and more recently through construction of semivariogram, which is more adequate and informative tool to measure structural variation of the variable of interest (Vieira et al, 1983). These methods of analysis have been used to investigate the spatial variability of several soil physical properties like hydraulic conductivity (Mulla, 1988; Rehfeldt et al., 1992; Romano, 1993); soil salinity (Hajrasuliha et al, 1980); soil water properties (Saddiq et al, 1985; Mulla, 1988; Davidoff & Salim, 1988, Berndtsson & Chen, 1994); infiltration rate (Vieria et al, 1981; Fonteh & Podmore, 1984); and soil chemical properties (Yost et al, 1982a, b; Tabor et al, 1984, Samra et al, 1988a,b; Berndtsson et al, 1993).

3.0 STUDY AREA

Jammu district falls along south western edge of the state of Jammu & Kashmir. Jammu district (Fig.1), with total geographic area of 3165 sq km has 1165 sq km of Siwalik hilly terrain and 2000 sq km of outer plain area comprising of Kandi and the Sirowal belts (Sangra, 1987). The study area comprises the outer plain area of Jammu district. The area is located at the foot of the outer most Siwalik hills and have an altitude varying between 280 to 400m above mean sea leval. The Kandi tract, having altitude between 320 to 400m above m.s.l., has steep topographic slopes. In this area, groundwater is very deep. In the Sirowal tract, the southern most plain tract of the district, the slopes are very gentle. Swampy conditions are found at places because of immense outflow of groundwater along the spring line marking the contact between the Kandi in the north and the Sirowal in the south.

The major natural drainage systems of the area, i.e. Chenab, Tawi, and Basantar river, enter the area from Siwalik hilly area. The chenab river is a perennial and snowfed originating from a vast watershed of Himachal Pradesh. The river Jammu Tawi flows through middle of Jammu City and falls into river Chenab about 16 km to the west of Jammu City. Many other seasonal nalas also traverse the study area.

The climate of the area is of subtropical type, characterised by three well defined seasons, viz., winter, summer and monsoon. The monsoon sets in by the beginning of July and continues till September. India Meteorological Department (IMD) has observed the normal annual rainfall at Jammu station about 1055 mm. Winter starts in November and continues till April, when summer season sets in. June is the hottest and January is the coldest month in the area. The average temperature in the Jammu region varies from 4 degree Celsius to 40 degree Celsius.

Outer plain area of the district is underlain by the sediments of the Recent-Sub-Recent times, laid down by the present day streams in the area. These sediments comprise of boulders, pebbles, gravels, sand and clay in highly mixed fashion near the hills and show stratification with distance from hills (Sangra, 1987).

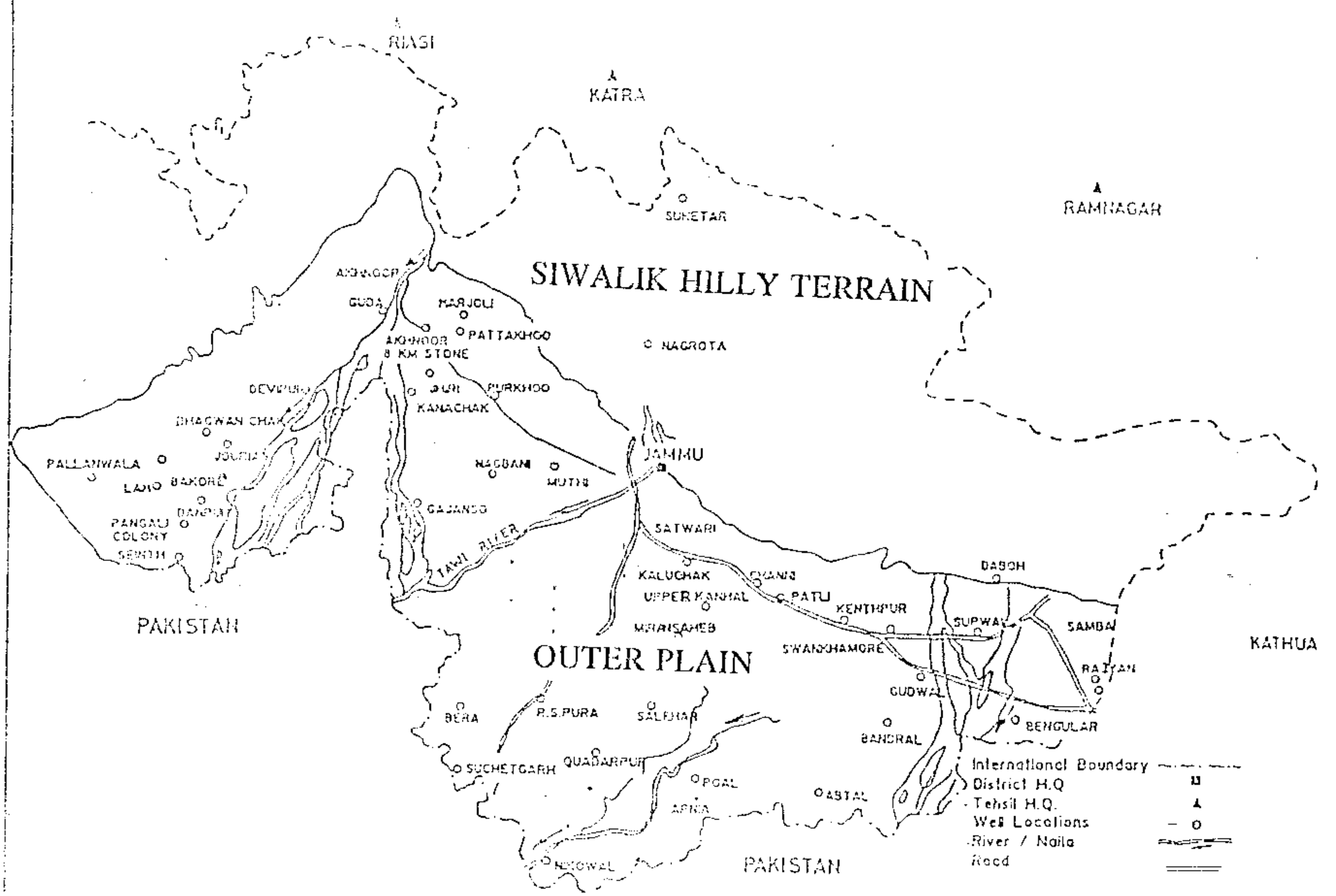


Fig. 4. Map of Jammu District

The Central Ground Water Board, Northern Western Himalayan Region, Jammu has carried out detailed investigation of various exploratory sites in J & K. The lithology of the Jammu Distt is generally non homogeneous and varies significantly from hilly areas to non hilly areas. The general lithostratigraphic sequence in the Kandi belt varies from clay to conglomerates with boulders, pebbles and gravels at many places. The general lithostratigraphic sequence in the plain areas of Jammu Distt (R S Pura) was found as multi coloured clay with little snad/silt and occasional clay hard plastic with gravels. The lithostratigraphic sequence as observed in a well at Akhnoor in Jammu Distt is given below.

Table-1 Lithostratigraphic Details for the Well at Akhnoor in Jammu District (Sangra, 1987)

Depth Range (m) below ground level	Thickness of Stratum (m)	Lithostratigraphic Sequence
G.L.- 14.64	14.64	Boulders large size with Clay
14.64- 18.00	3.36	Conglomerate Soft
18.00- 19.52	1.52	Conglomerate Soft
19.52- 23.79	4.27	Conglomerate Hard
23.79- 28.06	4.27	Conglomerate Hard
28.06- 32.02	3.96	Conglomerate Soft
32.02- 38.12	6.10	Conglomerate Hard
38.12- 43.61	5.49	Conglomerate Hard
43.61- 87.23	43.62	Conglomerate with Sand fine to coarse grained Conglomerate

Groundwater in the outer plain region occurs both under water table and confined conditions in the Sirowal and under unconfined conditions in the Kandi belt. In Sirowal zone the water table is shallow, whereas in Kandi belt the water level is very deep. Rainfall is the main source of replenishment to groundwater body in the area, in addition to influent seepage from nalla beds. The flow direction of the groundwater is broadly from north to south and correspond roughly with the topographic slope.

The soils of Jammu region show a great heterogenity. The soils of the foot hills, and areas adjacent to them, comprises of loose boulders and gravel with ferruginous. The plains of Jammu district are of alluvial nature. The Soil Survey Organisation, Department of Agriculture, Jammu has classified soils of R.S. Pura tehsil as Langotian soil, Bansultan soil and Kotli soil.

The soils of Langotian series are deep to very deep, varying in colour from light yellow to dark brown and grayish brown. Soils are of silty loam to silty clay loam texture in the lower layers, the texture varies between silty clay loam to silty clay. The permeability within the profile is low. The soils of Bansultan series are deep to very deep, with dark brown to yellowish brown colour. The texture varies between sandy loam to silt loam. These are well drained soils with moderate permeability. The soils of Kotli series are very deep, medium to heavy textured having impeded drainage. Permeability within the profile is moderately low to moderate.

On the basis of chemical characteristics of groundwater observed at 47 observation wells in the Jammu Distt in August 1994, December 1994 and March 1995, it was concluded that the quality of ground water varies from place to place (Jain et. al. 1994-95). Higher values of certain constituents at certain locations indicated that the water should not be used for domestic purposes without thorough analysis.It was also found that the groundwater can be used for irrigation on almost all soils with little danger of the development of harmful levels of exchangeable sodium. It was also found that groundwater at village Suchetgarh, Arnia and Gajansoo falls under very high salinity zone in the U.S. Salinity Laboratory classification and so should not be used for irrigation purpose.

4.0 METHODOLOGY

The theory of regionalised variables has been used in this report for quantifying the spatial variability and to interpolate the groundwater quality data. A brief theory of the technique is given in the following pages.

When a variable is distributed in space, it is said to be "regionalized" (Journel and Huijbregts, 1978). A Regionalized Variable (ReV) is defined by Matheron (1963) as the variable that spreads in space and exhibits certain spatial structure. Such variables show a complex behaviour. Their variations in space are erratic and often unpredictable from one point to another; however, these are not completely random as these exhibit some spatial correlation. So, a ReV is simply a function of space, but generally a very irregular function. All the parameters generally used in groundwater hydrology, such as transmissivity, hydraulic conductivity, piezometric heads, precipitation, vertical recharge, quality etc. can be called regionalized variables.

Let Z be a property of the aquifer, say the piezometric level; then, $Z(x)$ is defined as a random function (RF) where x represents the coordinates in 1.2, or 3 dimensions. $z_1(x)$ is called a realization of $Z(x)$. The function Z can have an infinite number of realizations i.e. $z_1(x)$, $z_2(x)$, $z_3(x)$......

The variogram, $2\gamma(h)$, is the arithematic mean of the squared difference between two experimental measures, $(Z(x_i)$ and $Z(x_i+h))$, at any two points separated by the vector h. For a set of N sample values,

$$2\gamma^*(h) = \frac{1}{N(h)} \sum_{i=1}^{N(h)} \left[z(x_i) - z(x_i+h)\right]^2 \tag{1}$$

Where,

N(h) is the number of experimental pairs separated by vector h in the data.

The semivariogram is calculated as

$$\gamma^{*}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[z(x_i) - z(x_i + h) \right]^2 \tag{2}$$

A plot of γ^*(h) versus the corresponding value of h, also called the semivariogram, is thus a function of the vector **h**, and may depend on both the magnitude and the direction of h. A sample plot of semivariogram is shown in Fig. 2.

The distance at which the variogram becomes constant is called the range, a. The value of the semivariogram at a distance equal to the range is called the sill. Often the sill is approximately equal to the sample variance (Journel and Huijbregts, 1978). The range, because it is expressed as distance, can be interpreted as the diameter of the zone of influence which represents the average maximum distance over which variables are related. At distance less than the range, measured properties of two samples become more alike with decreasing distance between them. Thus, the range provides an estimate of areas of similarity. Semivariograms may also increase continuously without showing a definite range and sill. Such types either correspond to RF which are only intrinsic or indicate the presence of non-stationarity. The value of the semivariogram at extremely small separation distance is called the nugget effect.

The behaviour of the semivariogram near the origin and at infinity are two other important features of the semivariogram as these express the qualitative characteristics of regionalization (Matheron, 1963). Behaviour near the origin characterizes the continuity of the ReV. The regularity of ReV is represented by the more or less regular behaviour of γ(h) near the origin. Examples of the four classical types of behaviour are shown in Fig. 3. Fig. 3(a) has a parabolic shape near the origin and presents a ReV with high continuity such a the head in a deep observation well as a function of time. (Delhomme, 1978). Linear shape (Fig. 3(b)) represents a ReV which has an "in average" continuity. Discontinuity at the origin corresponds to a variable presenting not even an "in average" continuity (Matheron, 1963). Two distinct points at a very close distance

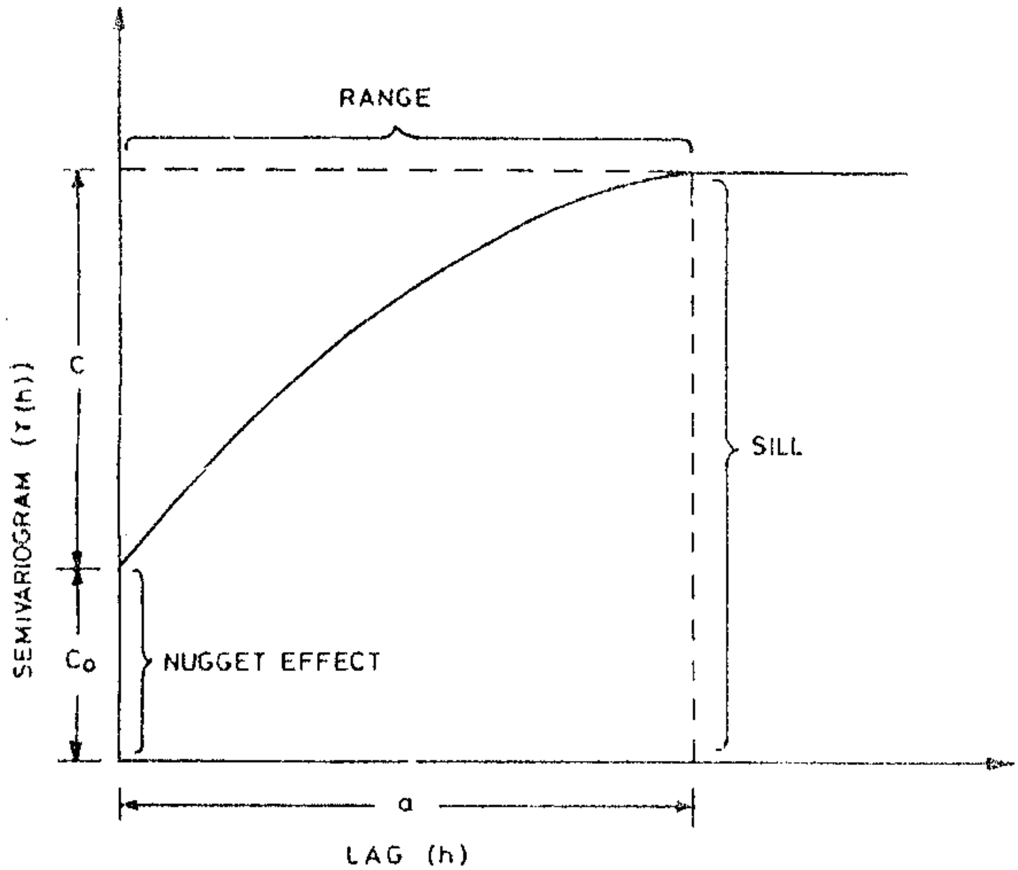


FIG. 2 PLOT OF SEMIVARIOGRAM

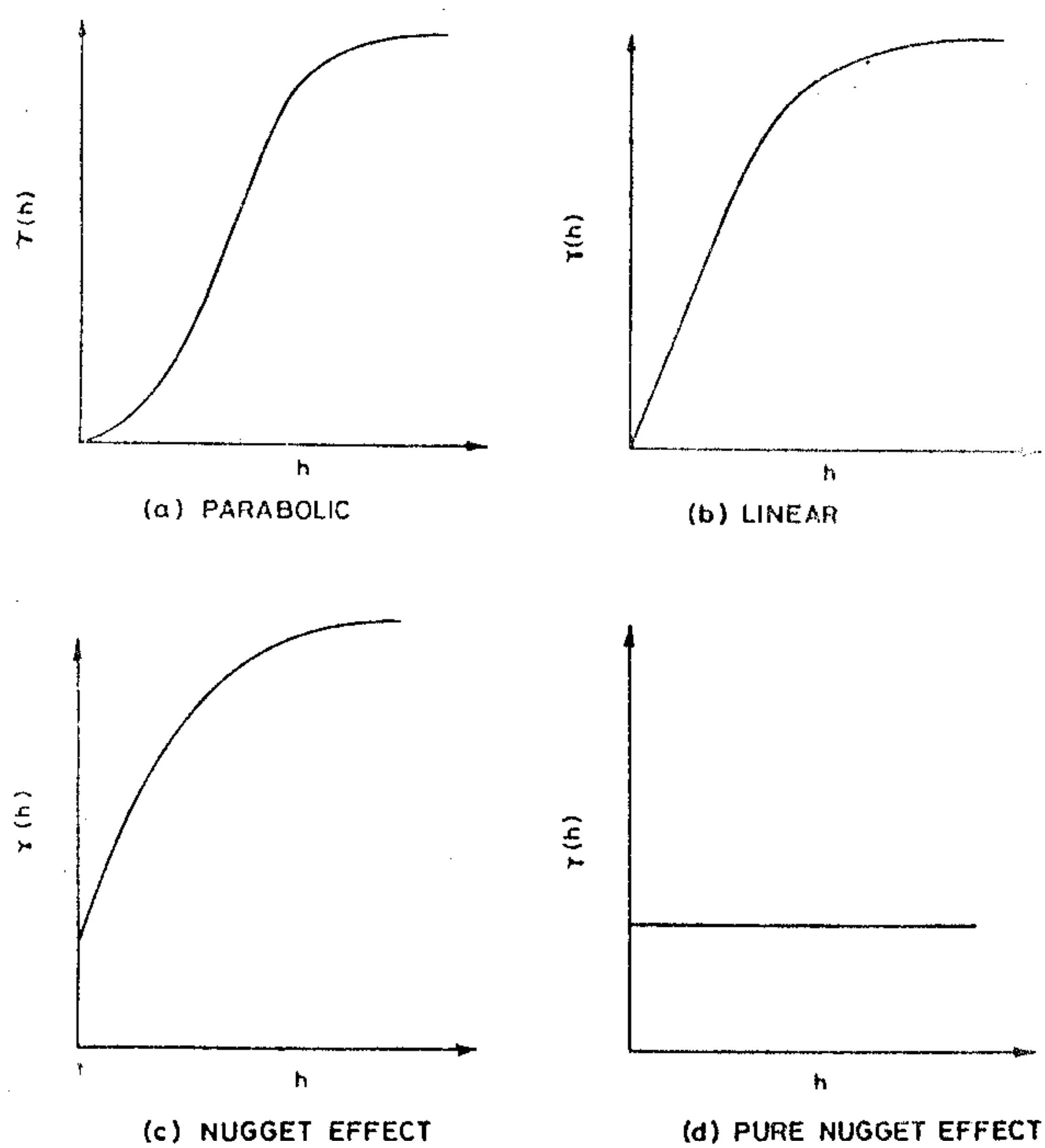


FIG. 3. BEHAVIOR OF THE SEMIVARIOGRAM NEAR THE ORIGIN

will also show a difference. Fig. 3(c) shows this nugget effect. Fig. 3(d) shows a pure nugget effect and it is the limit case when the semivariogram appears solely as a discontinuity at the origin (Journel and Huijbregts, 1978). It corresponds to a total absence of auto correlation and hence pure random behaviour.

The semivariogram increases more slowly as lag distance 'h' tends to infinity. An experimental semivariogram, which increases at least as rapidly as $|h|^2$ for large 'h' does not hold the intrinsic hypothesis and indicates non-stationarity (Journel and Huijbregts, 1978).

The semivariogram, given in Eq.2 is also termed the true semivariogram of the ReV. As only one realization of the RF is available, the true semivariogram can only be estimated and this estimate is known as the experimental semivariogram.

If the sampling is done on a regular grid, the $\gamma^*(h)$ may be estimated for values of h, known as lag distance or lag increment which are multiples of the grid spacing. This situation is rare in practice, particularily in the context of groundwater and the chance of finding pairs at exactly same specified distance h is very small. To overcome this, a tolerance, δh is placed on the distance. Every pair of observations that are separated by a lag $h \pm \delta h/2$ are then used to estimate $\gamma^*(h)$.

The above procedure is used for calculating the isotropic experimental semivariogram, also known as omnidirectional semivariogram. In this case, it is assumed that the variation is the same in every direction. To find the anisotropies, the semivariograms are calculated in different directions. To do this, a tolerance, $\delta\theta$, is placed on the directional angle.

The experimental semivariogram has discrete values and irregular shape due to the limited sampling. A mathematical function used to approximately represent this semivariogram is known as the theoretical semivariogram. The process of fitting a theoretical model to experimental semivariogram is called structural analysis. This process is the first and most important step in the geostatistics as it affects the final results.

For any function to be a valid function for a semivariogram, it should meet the positive definite condition. It is safe to use only those functions which are tested and are used in literature. These functions are varied enough to enable a satisfactory fit to all sample variograms likely to be encountered in practice. Some of these models are discussed below.

Spherical model :- (Fig. 4(a))

$$\gamma(h) = \begin{cases} C_0[1 - \delta(h)] + C\left[\dfrac{3}{2}\dfrac{h}{a} - \dfrac{1}{2}\dfrac{h^3}{a^3}\right] & h \le a \\ C_0 + C & h > a \end{cases} \tag{3}$$

Exponential model :- (Fig. 4(b))

$$\gamma(h) = C_0[1 - \delta(h)] + C\left[1 - \exp(-\frac{h}{a})\right] \tag{4}$$

Gaussian model :- (Fig. 4(c))

$$\gamma(h) = C_0[1 - \delta(h)] + C\left[1 - \exp(-\frac{h^2}{a^2})\right] \tag{5}$$

Linear model :- (Fig. 4(d))

$$\gamma(h) = C_0[1 - \delta(h)] + bh \tag{6}$$

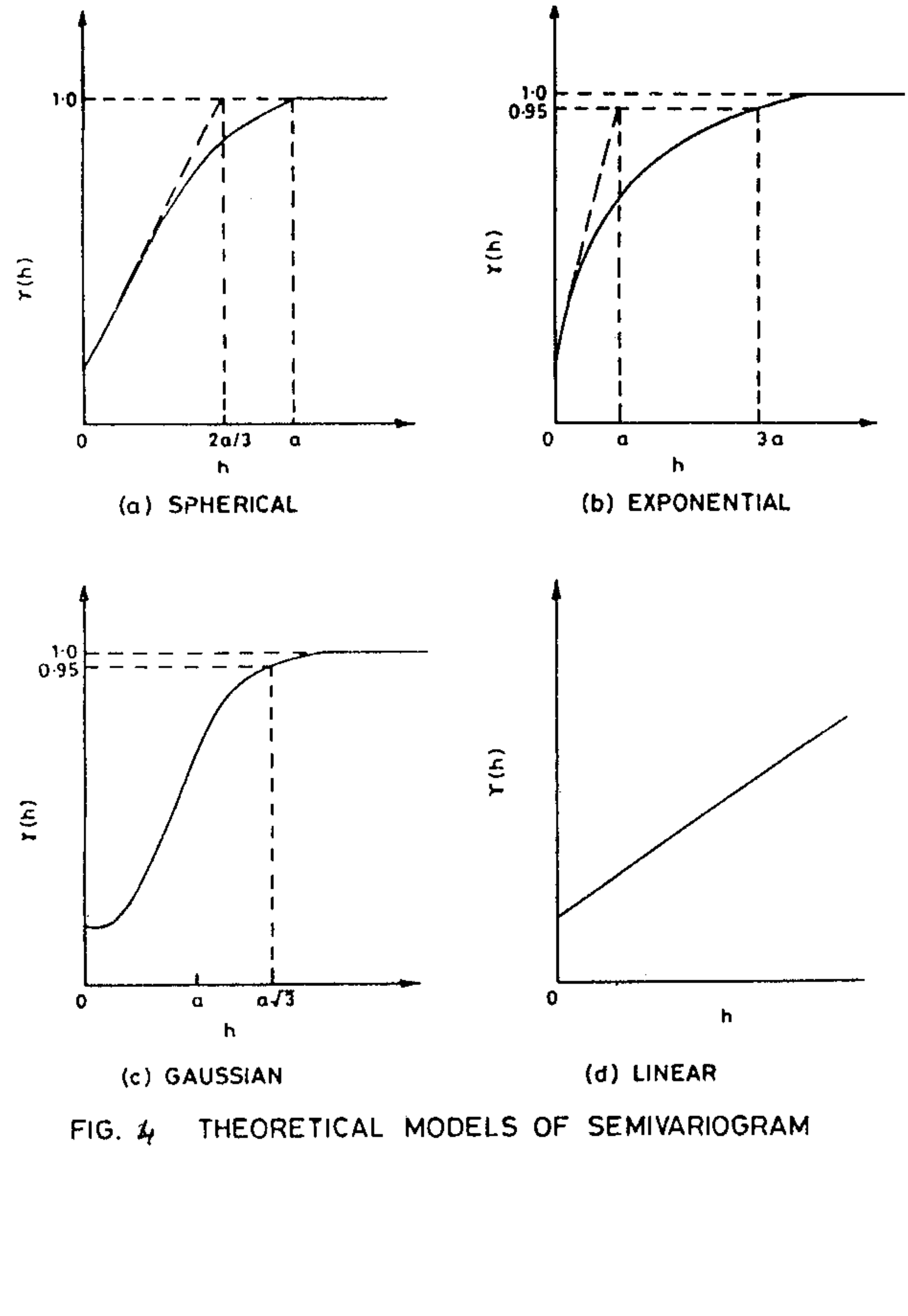


FIG. 4 THEORETICAL MODELS OF SEMIVARIOGRAM

Where,

$$\delta(h) \text{ is the Kronecker delta} = \begin{cases} 1 & h = 0 \\ 0 & h \neq 0 \end{cases}$$

C_0 is the Nugget effect

C_0+C is the Sill

and a is the Range and b is the slope at the origin.

4.1 Kriging

Kriging is a technique of making optimal, unbiased estimates of regionalized variables at unsampled locations using the structural properties of the semivariogram and the initial set of data values. Kriging can be applied to estimate the value of a variable at a particular point (Punctual Kriging) or to estimate the average value of a block (Block Kriging). In punctual kriging, by changing the position of the point, it is possible to estimate the whole area of interest. Only the sample locations which are spatially related to the kriged locations (i.e. within the range of spatial dependence) are used in kriging.

Consider a situation in which a property is measured at a number of points, x_i, within a region to give values of $z(x_i)$, $i=1,2,3,\ldots,N$. (x_i is the coordinate of the observation point in 1, 2 or 3-dimensional space). From these observations, the value of the property at any place x_0 can be estimated. The place might be a 'point', i.e. an area of the same size and shape as those on which measurements were made, or a larger area or block. Such situations commonly arise in hydrology. For example, in the estimation of rainfall, temperature, sunshine; in the estimation of hydrological parameters such as transmissivity, piezometric head, solute concentration in a plume etc.

Linear geostatistics estimates the kriged value of z at x as the weighted sum of the measured values i.e. for point estimation.

$$z^*(x_0) = \sum_{i=1}^{N} \lambda_i z(x_i) \qquad i=1,2,3,...,N \tag{7}$$

where,

$z^*(x_0)$ = estimated value at x_0

λ_i = weights chosen so as to satisfy suitable statistical conditions

$z(x_i)$ = observed values at points x_i

Apart from providing the estimate of a property, geostatistics also provide the estimation variance which measures the accuracy of the estimate.

By taking $z(x_i)$ as a realization of the random function $Z(x_i)$ and so $z(x_0)$ as the realization of $Z(x_0)$, the Eq.7 can be written as

$$Z^*(x_0) = \sum_{i=1}^{N} \lambda_i Z(x_i) \qquad i=1,2,3,...,N \tag{8}$$

In kriging, the weights λ_i are calculated so that $Z^*(x_0)$ is unbiased and optimal.

$$E\{Z^*(x_0) - Z(x_0)\} = 0 \tag{9}$$

The condition of optimality means that the variance of the estimation error should be minimum i.e.

$$Var\{Z^*(x_0) - Z(x_0)\} = minimum \tag{10}$$

substituting Eq.8 into Eq.9 leads to

$$\sum_{i=1}^{N} \lambda_i = 1 \tag{11}$$

for stationarity of order-2. The estimation will be unbiased if the Eq.11 will hold.

Substitution for $Z^*(x_0)$ in the minimum variance condition and rearrangement of resulting terms in terms of $\gamma(h)$ and $\gamma(0)$ yield :

$$E\left\{\left[Z^*(x_0) - Z(x_0)\right]^2\right\} = -\sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_i \lambda_j \gamma(x_i,x_j) + 2\sum_{i=1}^{N} \lambda_i \gamma(x_i,x_j) \tag{12}$$

The Eq.12 is a quadratic function of the weights λ_i. The minimization of the above function, subject to the linear constraint of Eq. 11 is found using the Lagrange multiplier, μ and taking the partial derivatives for all λ_i, i.e.

$$\frac{\delta}{\delta \lambda_i}\left[E\left\{\left[Z^*(x_0) - Z(x_0)\right]^2\right\} - 2\mu\left(\sum_{i=1}^{N} \lambda_i - 1\right)\right] = 0 \tag{13}$$

Where,

μ = Lagrangian multiplier

On simplification of Eq. 13

$$-2\sum_{j=1}^{N} \lambda_j \gamma(x_i,x_j) + 2\gamma(x_i,x_0) - 2\mu = 0 \tag{14}$$

Rearranging and combining with Eq. 11 results in the kriging system equations.

$$\begin{aligned} &\sum_{j=1}^{N} \lambda_j \gamma(x_i,x_j) + \mu = \gamma(x_i,x_0) \qquad i = 1,2,3,\ldots,N \\ &\sum_{i=1}^{N} \lambda_i = 1 \end{aligned} \tag{15}$$

Substitution of Eq. 15 into Eq.12 yields the estimation variance, $\sigma_k^2(x_0)$ at x_0, as:

$$\sigma_k^2(x_0) = \sum_{i=1}^{N} \lambda_i \gamma(x_i,x_0) + \mu \tag{16}$$

Solution of the above set provides the values of λ_i which can be used with Eq. 7 for estimation. The kriged estimates were thus obtained as a linear sum of products of observed values and their respective weights λ_i. The resulting interpolator thus depends on the data values and also on their geometric location or data configuration through semivariance. This unique quality of the estimator distinguishes it from other conventional estimators. The formulation of kriging estimation does not depend on any particular sample probability density distribution. However, if data are clearly lognormal the logarithm of the observations, $y(x_i)$ can be estimated with a lower estimation variance than the raw data $z(x_i)$ (Journel and Huijbregts, 1978). i.e.

$$y(x_i) = \ln (z(x_i)) \tag{17}$$

The procedure requires an additional final step to revert to the original regionalized variable space. Should the estimated value by ordinary kriging of a transform be $Y^*(x_0)$ with estimation variance $\sigma_k^2(x_0)$, then the kriging estimate $Z^*(x_0)$ for the original variable is

$$Z^*(x_0) = e^{Y^*(x_0)+0.5\sigma_k^2(x_0)} \tag{18}$$

The kriging technique developed for the estimation of non-stationary ReV is called universal kriging. There are many other variations in kriging such as cokriging and disjunctive kriging. However, there are not used in the present study, and hence are not presented herein.

5.0 RESULT AND DISCUSSION

The methodology described above has been applied to the groundwater quality data of Jammu district of J&K state (India). Fig.5 shows the location of 44 observation points (for which groundwater quality data is available) for December 1994, with the number identifying well identity. The data pertains to the study conducted by Jain et. al. (1994-95) in the Jammu district of J&K state for evaluating the quality of the groundwater of the area. In the above study, the samples were collected from the open wells in August 1994, December 1994 and March 1995, and physico-chemical analysis was conducted on these samples. From the study, it was concluded that the quality of ground water varies from place to place. It was recommended that any water source must be thoroughly analysed and studied before being used for domestic applications. It was also concluded that ground water at few places falls under very high salinity zone and so should not be used for irrigation purposes.

The basic statistics, pertaining to the data, such as the mean, variance (s^2), coefficient of variance (CV), standard deviation (s), skewness, kurtosis, median and minimum and maximum value of the observed groundwater quality are shown in Table 2. The pH ranges from 6.738 to 8.128 with a mean of 7.315, conductance ranges from 0.34 mmhos/cm to 3.5 mmhos/cm with a mean of 1.126 mmhos/cm, alkalinity ranges from 152.0 mg/L to 808.0 mg/L with a mean of 380.636 mg/L, chloride ranges from 1.0 mg/L to 520.0 mg/L with a mean of 83.636 mg/L, total hardness ranges from 168.0 mg/L to 916.0 mg/L with a mean of 415.182 mg/L, calcium ranges from 36.89 mg/L to 279.1 mg/L with a mean of 115.798 mg/L, magnesium ranges from 1.0 mg/L to 102.06 mg/L with a mean of 30.809 mg/L, sodium ranges from 3.0 mg/L to 350.0 mg/L with a mean of 65.593 mg/L and potassium ranges from 0.80 mg/L to 282.0 mg/L with a mean of 39.536 mg/L. Potassium has the maximum percentage coefficent of variation (% C.V.) of 191.857 % whereas pH has the minimum i.e. 3.832%.

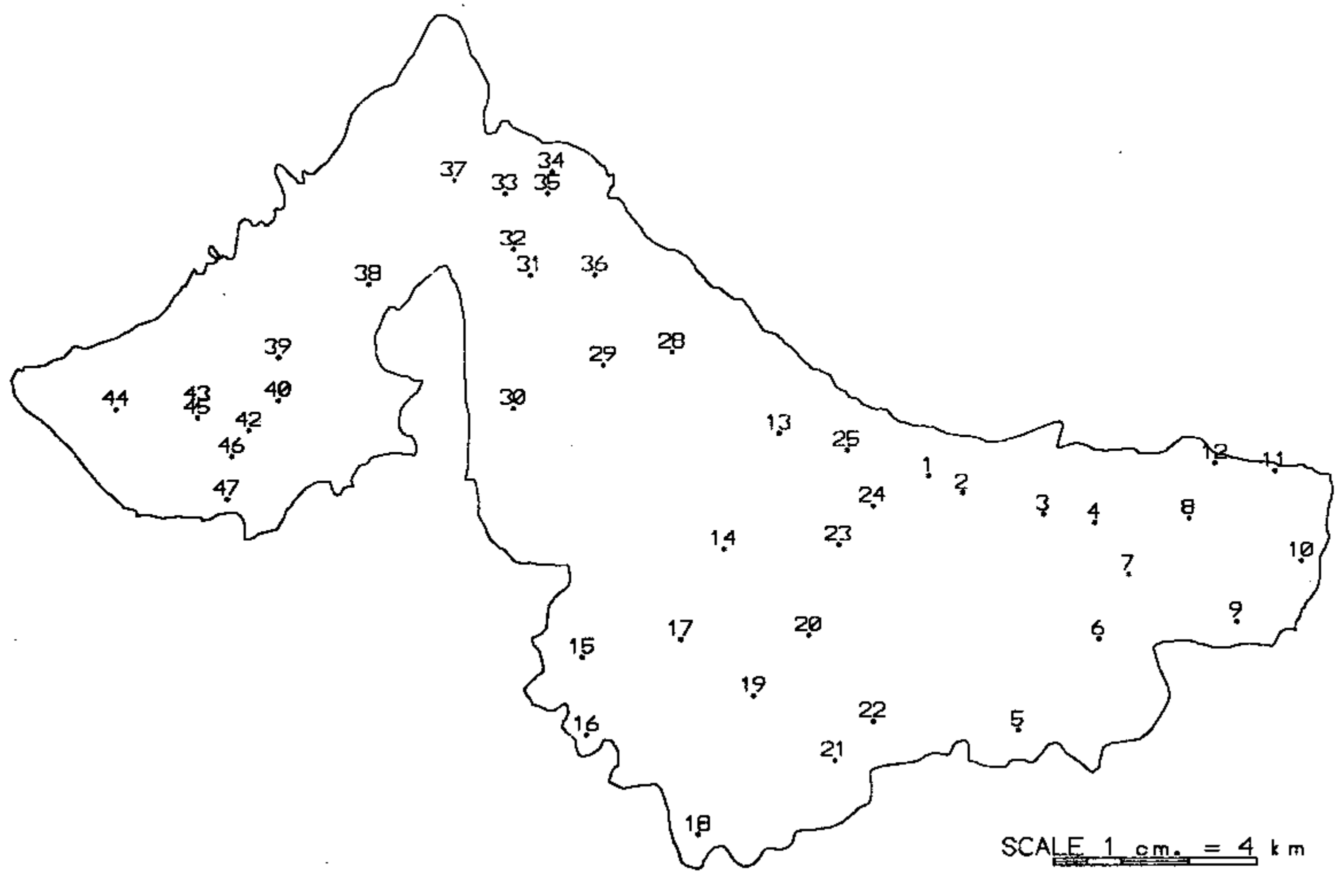


Fig.5. Location of observation points

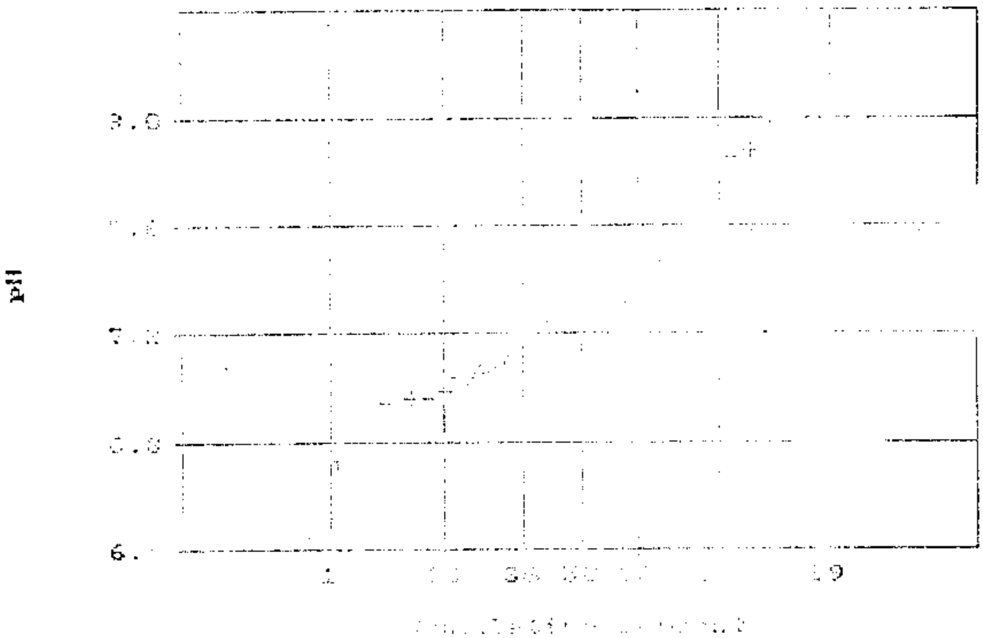
Table 2 Basic statistics of data set

Sr. No	Parameter	pH	Cond. *(mmhos/cm)*	Alkal *(mg/L)*	Chloride *(mg/L)*	Hardness *(mg/L)*	Calcium *(mg/L)*	Magnesium *(mg/L)*	Sodium *(mg/L)*	Potassium *(mg/L)*
1.	No. of data	44	44	44	44	44	44	44	44	44
2.	Mean	7.315	1.126	380.636	83.636	415.182	115.798	30.809	65.593	39.536
3.	Variance	0.079	0.595	26154.38	13439.31	31041.92	3602.138	479.886	6341.54	5753.71
4.	% C. V.	3.832	68.506	42.488	138.618	42.436	51.83	71.104	121.406	191.857
5.	Std devi	0.280	0.771	161.723	115.928	176.187	60.018	21.906	79.634	75.853
6.	Skewness	0.821	1.632	0.762	2.220	1.125	1.213	1.033	2.046	2.232
7.	Kurtosis	3.955	5.242	2.72	7.878	3.784	3.598	4.113	6.743	6.509
8.	Median	7.280	0.845	338.0	28.0	356.0	102.66	26.245	35.0	6.0
9.	Min value	6.738	0.340	152.0	1.0	168.0	36.89	1.0	3.0	0.80
10.	Max value	8.128	3.500	808.0	520.0	916.0	279.1	102.06	350.0	282.0

As stated earlier in chapter 3, the formulation of kriging estimation does not depend on any particular sample probability density distribution. However, if data are clearly lognormal, the log-transformation of data produces better results. Fig. 6 to 14 shows the fractile diagram, based on the cumulative frequency distribution, of the original and logarithm of various groundwater variables. Fractile diagram of pH (Fig. 6) yields almost a straight line indicating that the pH is normally distributed. Fig. 6 shows that the log-transformed data of pH does not improve the straight line fit in the fractile diagram. So, pH can be considered as normally distributed. The Fractile diagrams of conductance (Fig. 7), alkalinity (Fig. 8), calcium (Fig. 9), sodium (Fig. 10), potassium (Fig. 11), magnesium (Fig. 12), chloride (Fig. 13), and total hardness (Fig. 14) does not represent a straight line, which indicates that these ground water quality parameters are not normally distributed. Fractile diagram based on the logarithm of these variables closely follow a straight line, except for magnesium and chloride. It indicate that the distribution of conductivity, alkalinity, calcium, sodium, potassium, and total hardness can be considrered as log-normal. As the distribution of magnesium and chloride is found not to be log-normal, the spatial variability of these parameters is analysed considering these parameters to be normally distributed. So, in this study, it was considered that pH, magnesium and chloride follow a normal distribution and all other parameters (conductivity, alkalinity, calcium, sodium, potassium, and total hardness) follow a lognormal distribution.

5.1 Semivariogram analysis

Experimental semivariograms are calculated for all the groundwater quality parameters for the December 1994 period. Only omnidirectional semivariograms have been calculated as the total number of observation points is small (only 44). As the observation points are not uniformly distributed in the study area, tolerance is introduced in distance. A lag distance of 4km and a distance tolerance of 2km are used to calculate the experimental semivariogram and the same is shown in Fig.15 to Fig.23. Note that the semivariograms for conductivity, alkalinity, calcium, sodium, potassium, and total hardness are calculated after log-transformation of the original data.



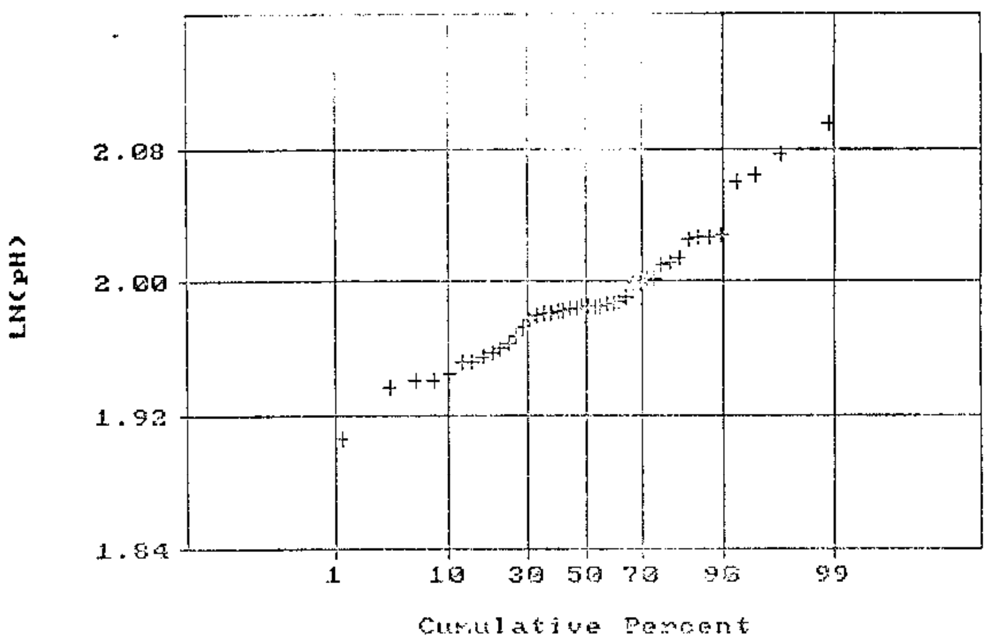


Fig. 6 . Fractile diagram of pH

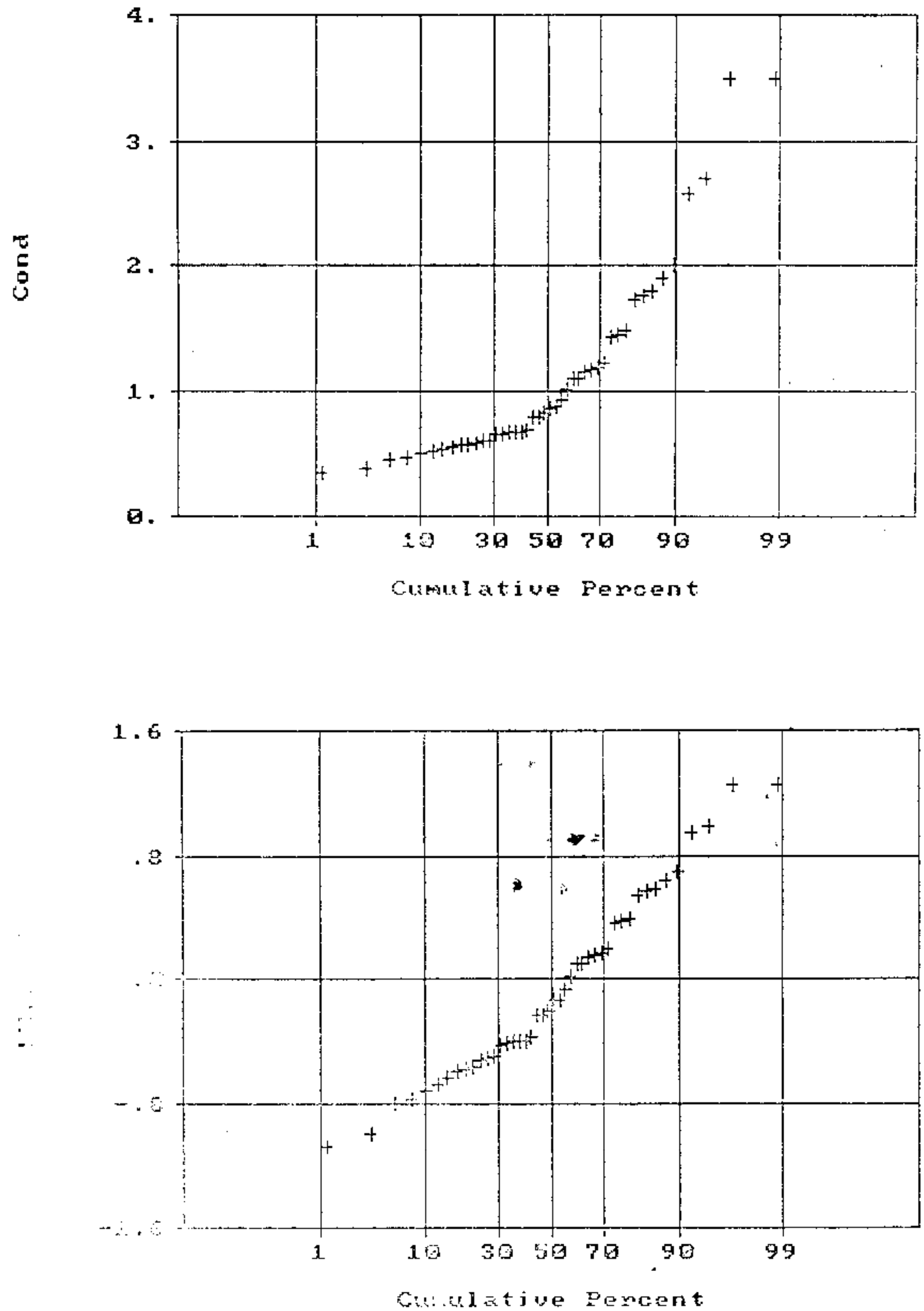


Fig. 9 . Fractile diagram of conductance

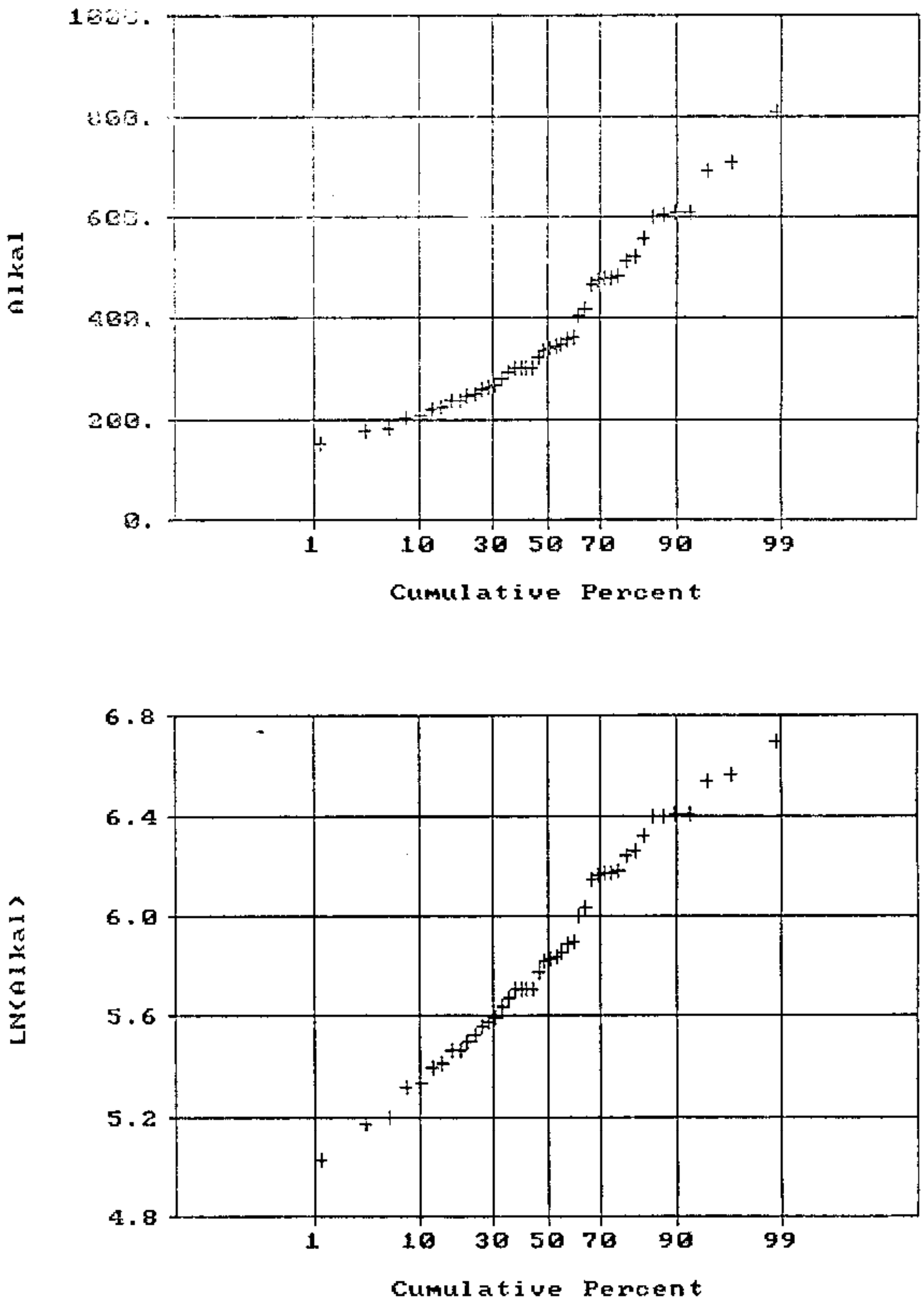


Fig. 8. Fractile diagram of alkalinity

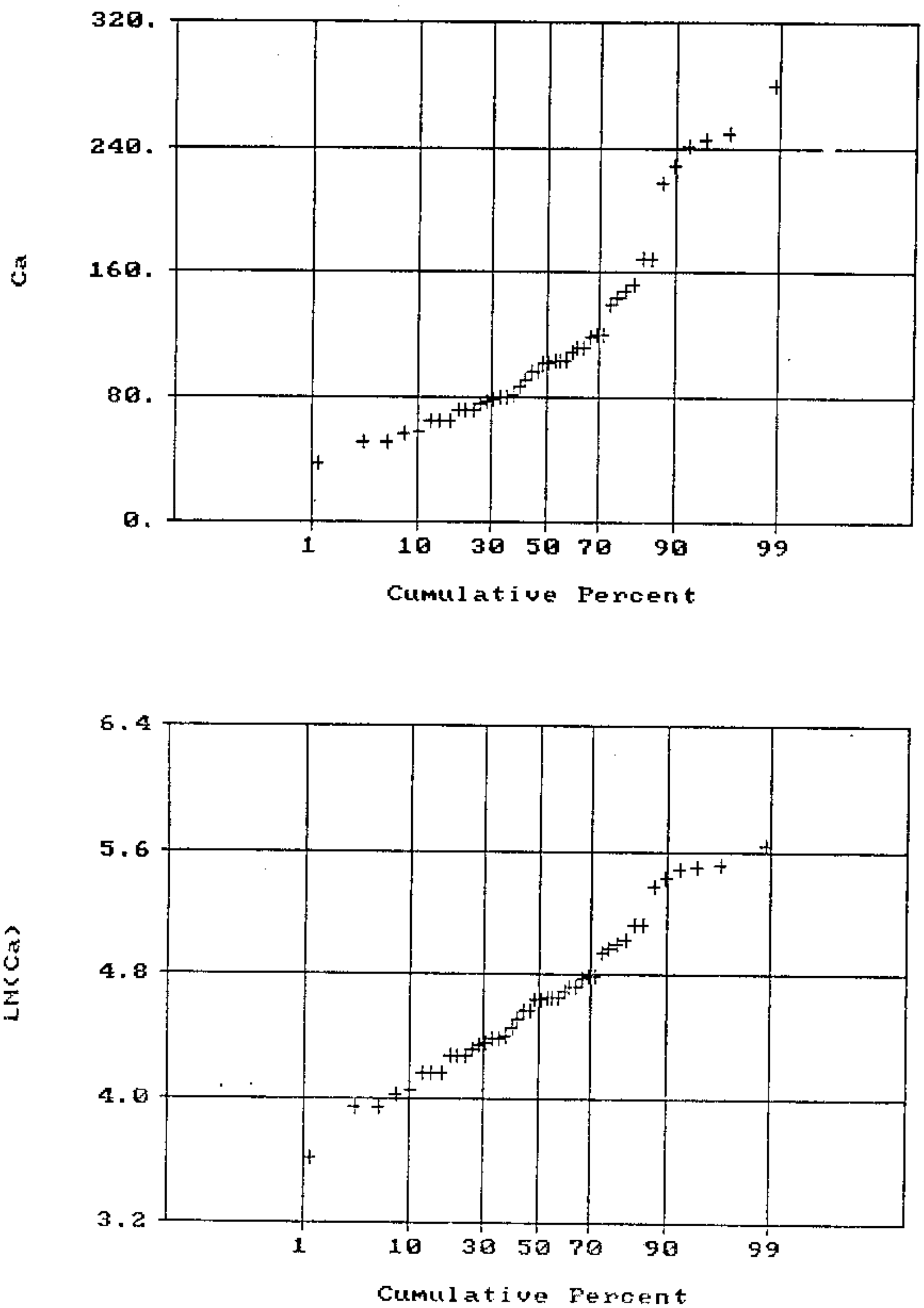


Fig. 5. Fractile diagram of calcium concentration

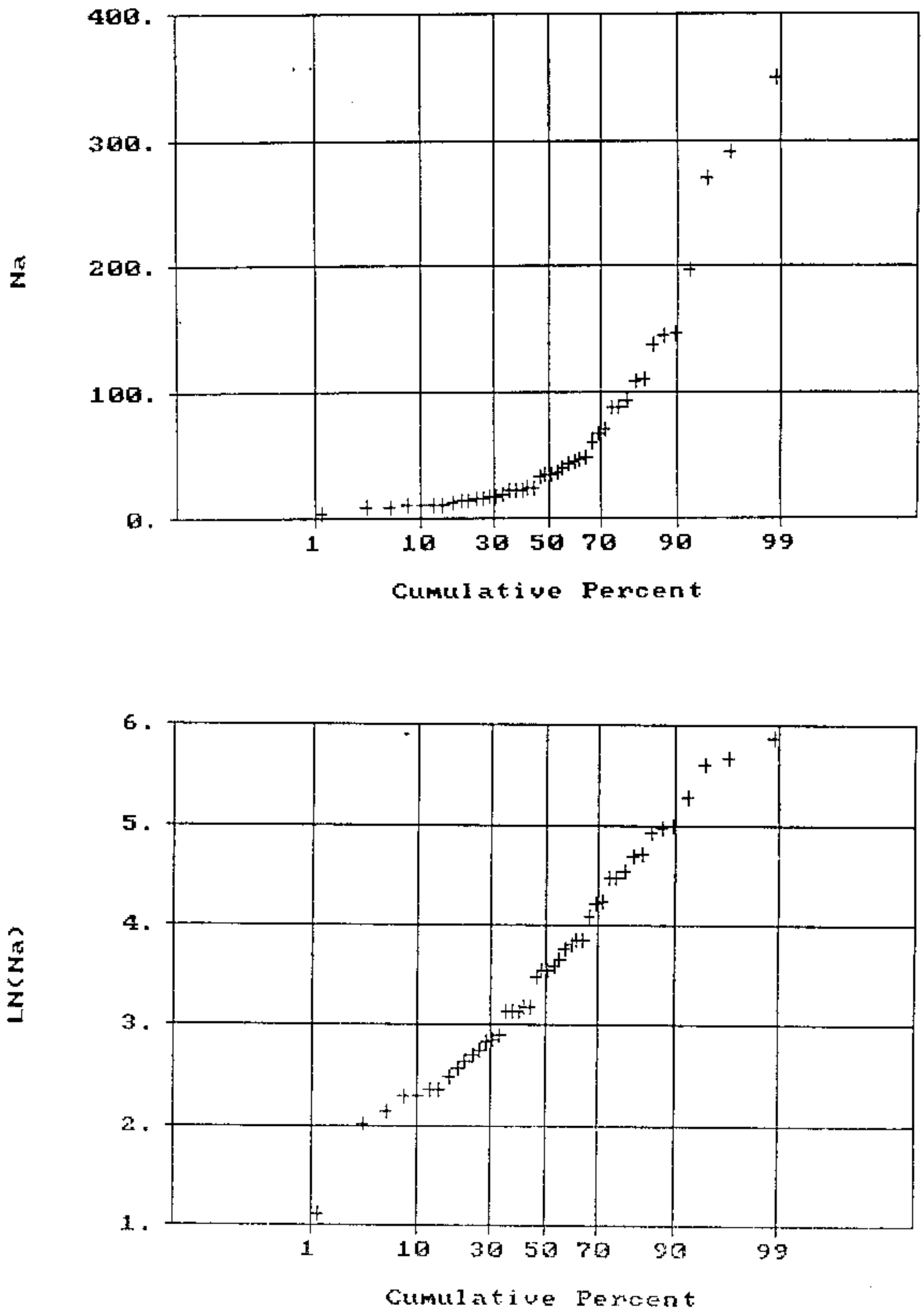


Fig. 10. Fractile diagram of sodium concentration

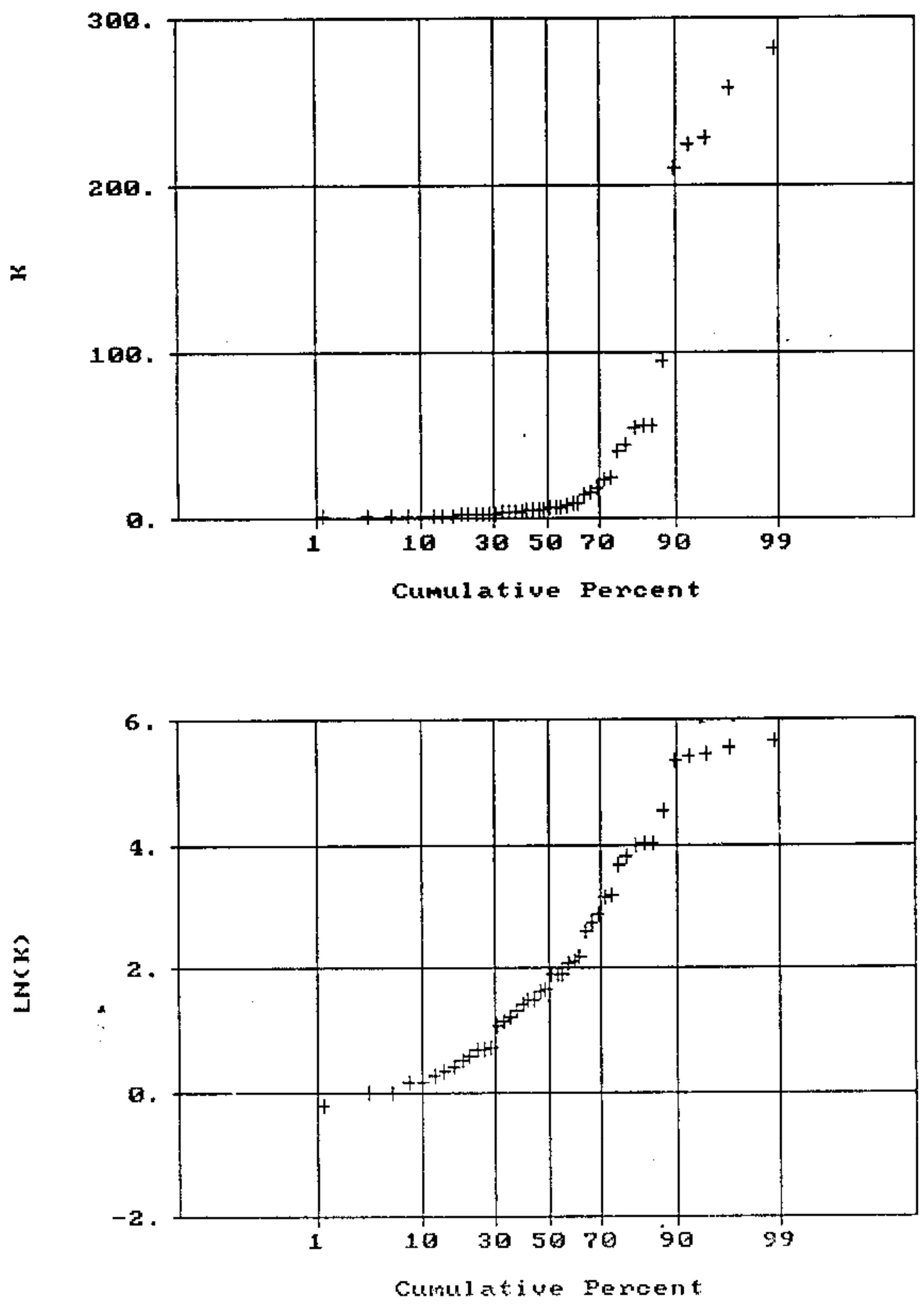


Fig. 11. Fractile diagram of potassium concentration

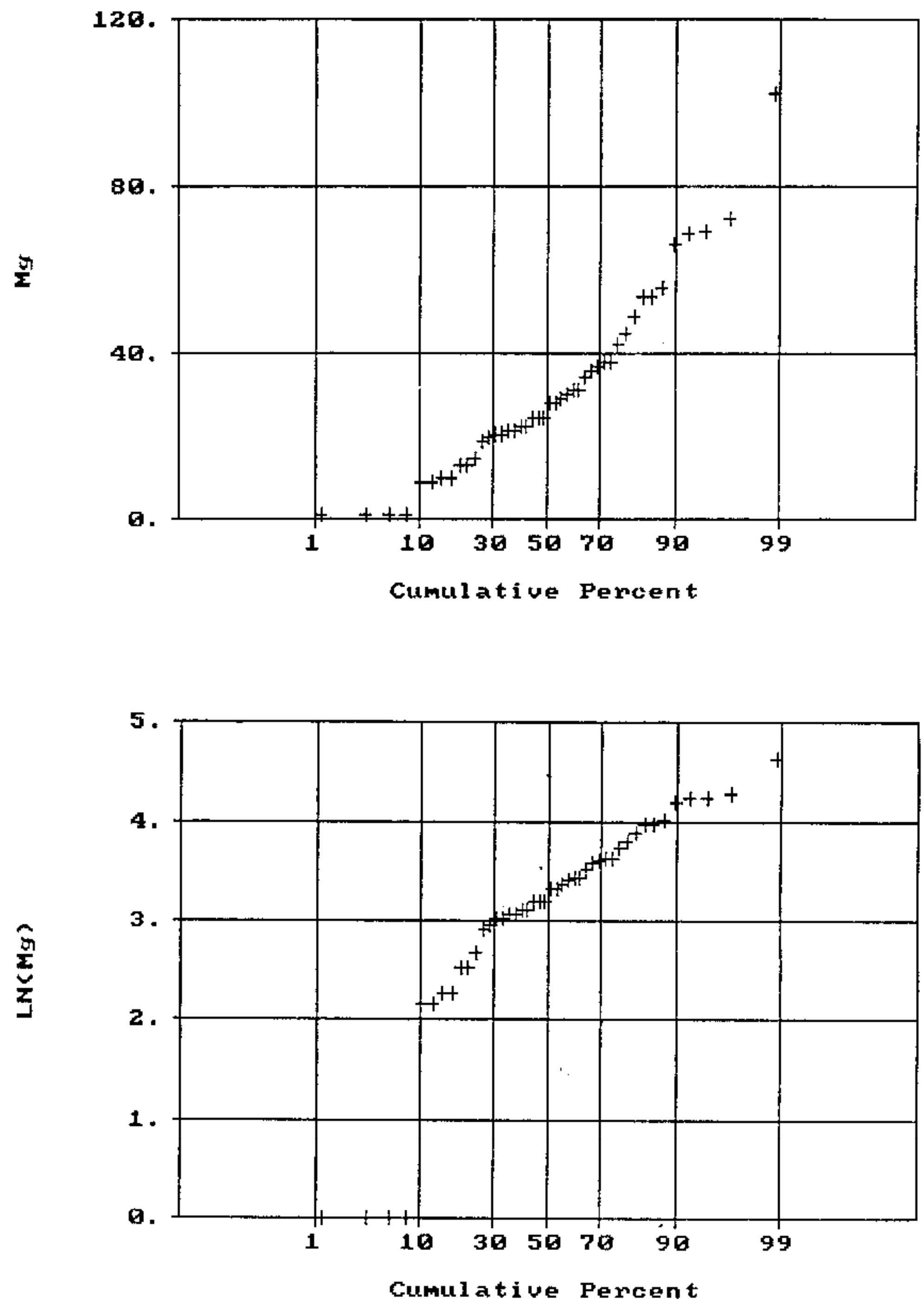


Fig. 12. Fractile diagram of magnesium concentration

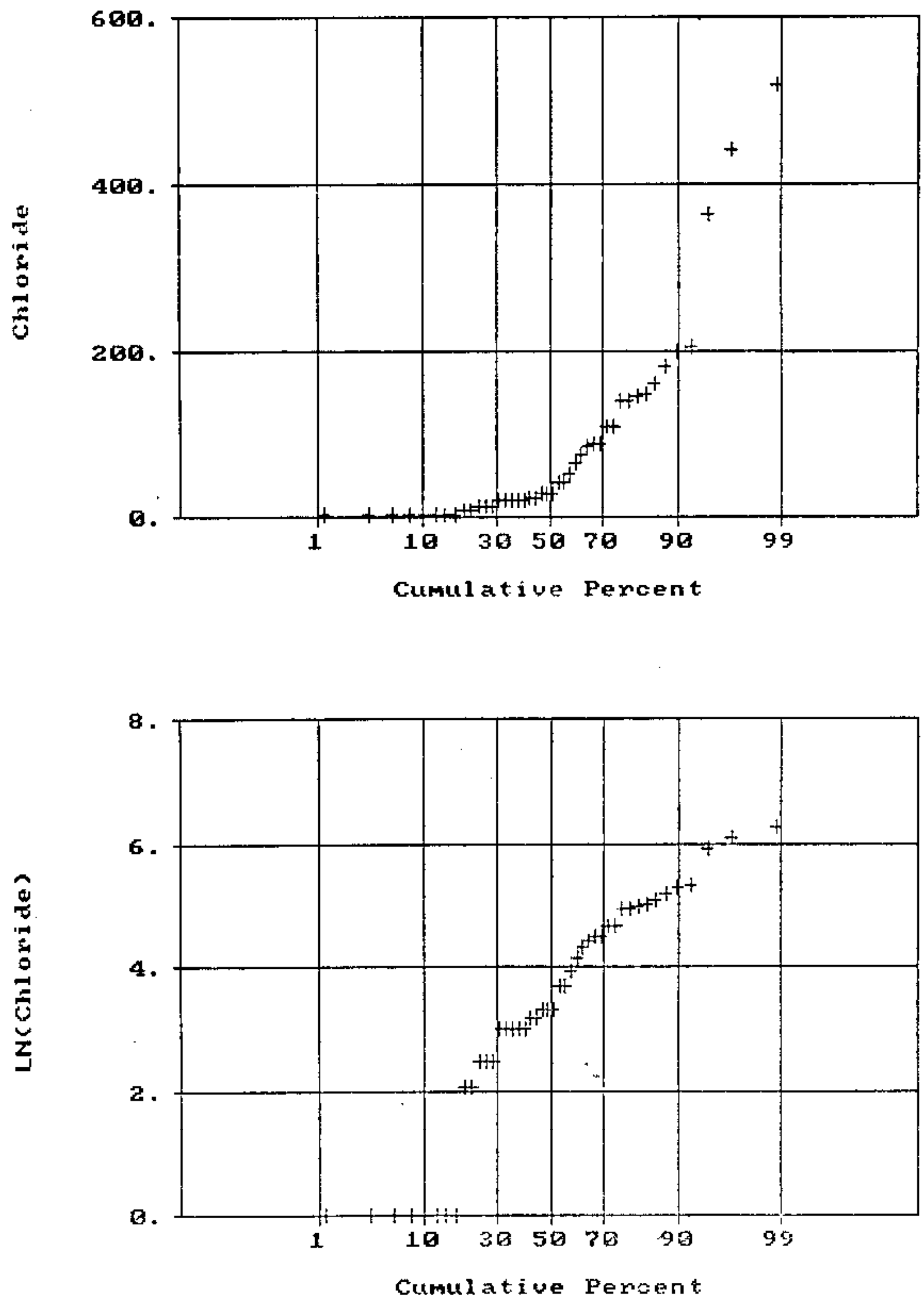
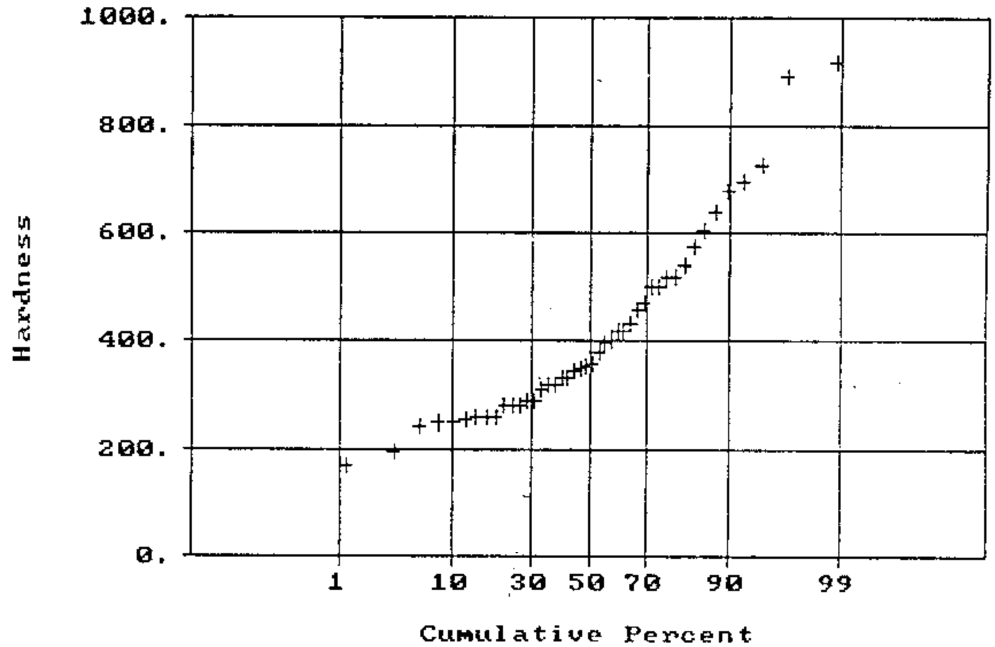


Fig. 13. Fractile diagram of chloride concentration



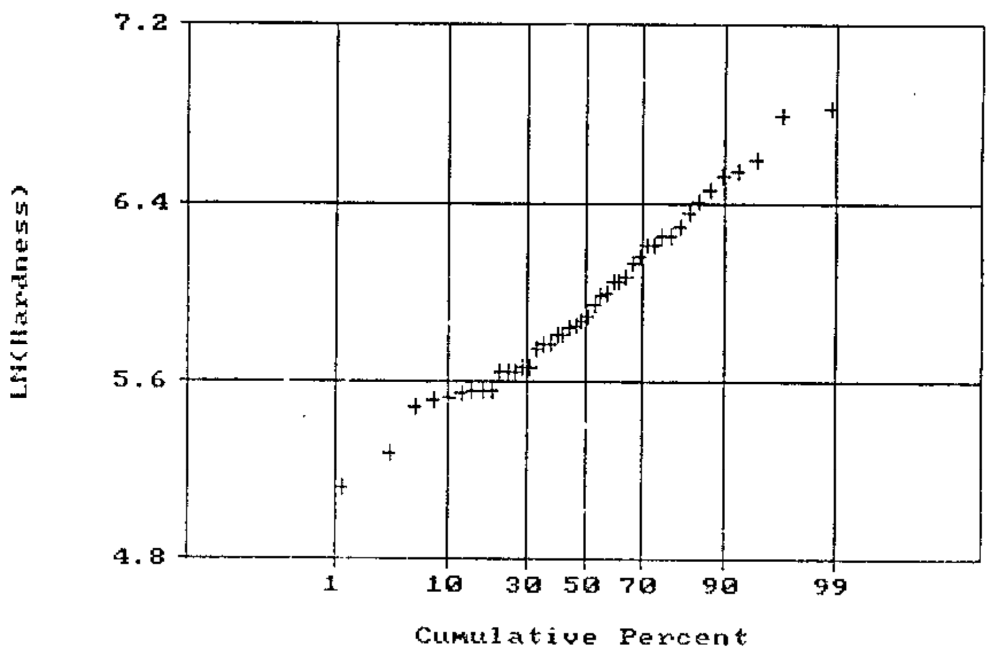


Fig.14. Fractile diagram of total hardness

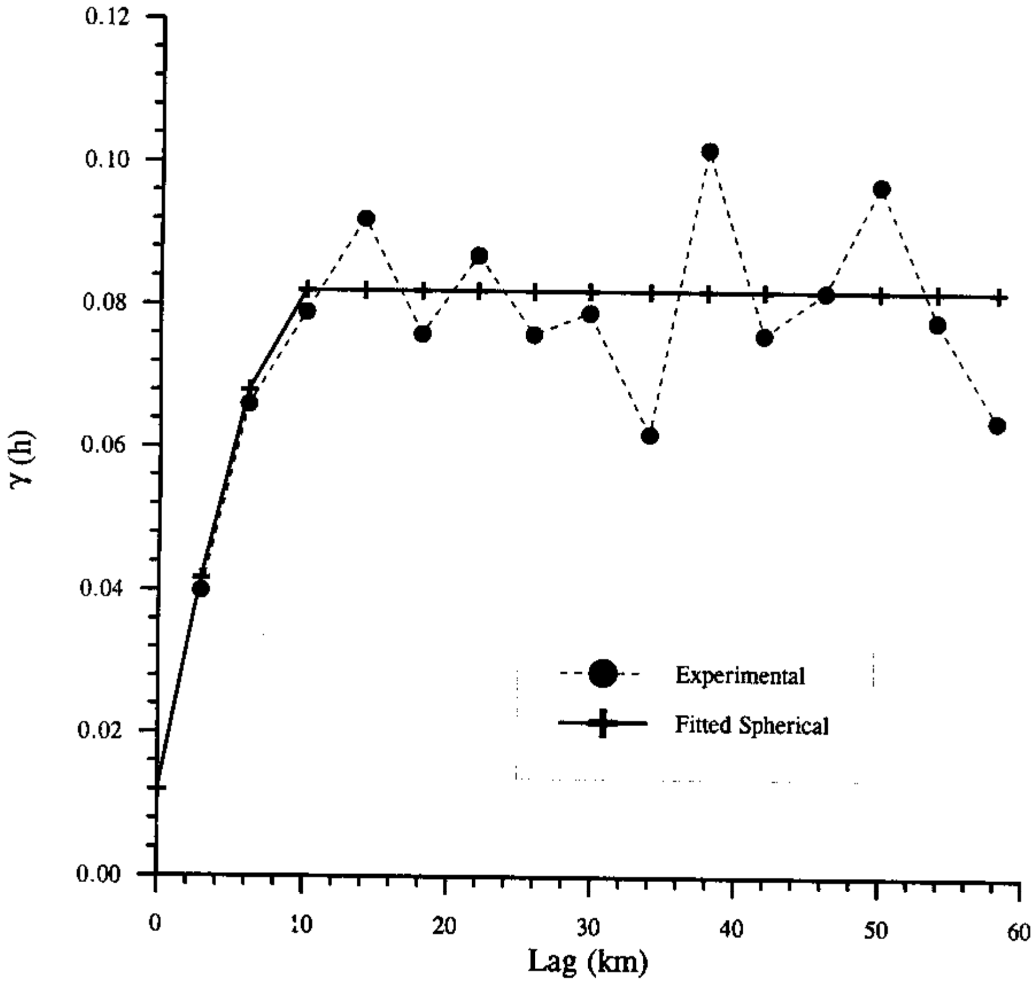


Fig. 15. Semivariogram of pH

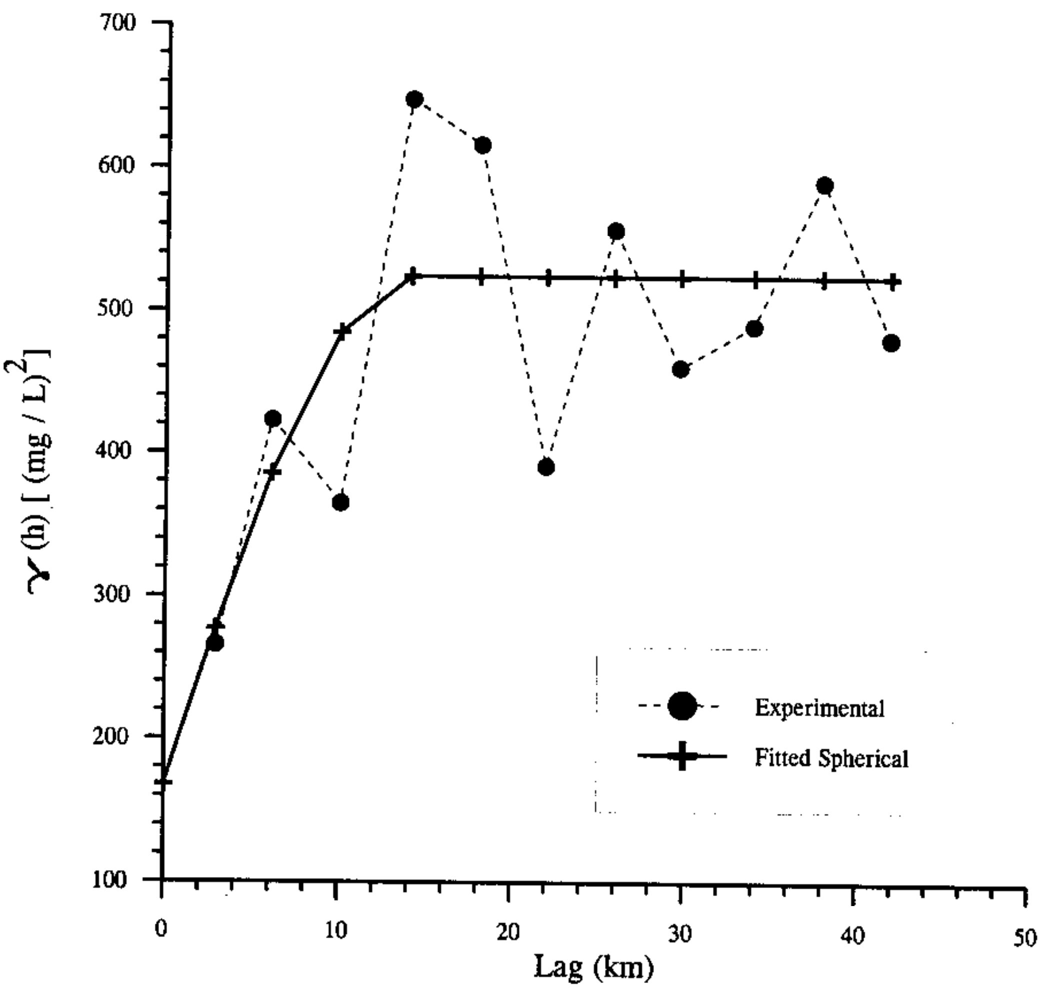


Fig. 16. Semivariogram of magnesium concentration

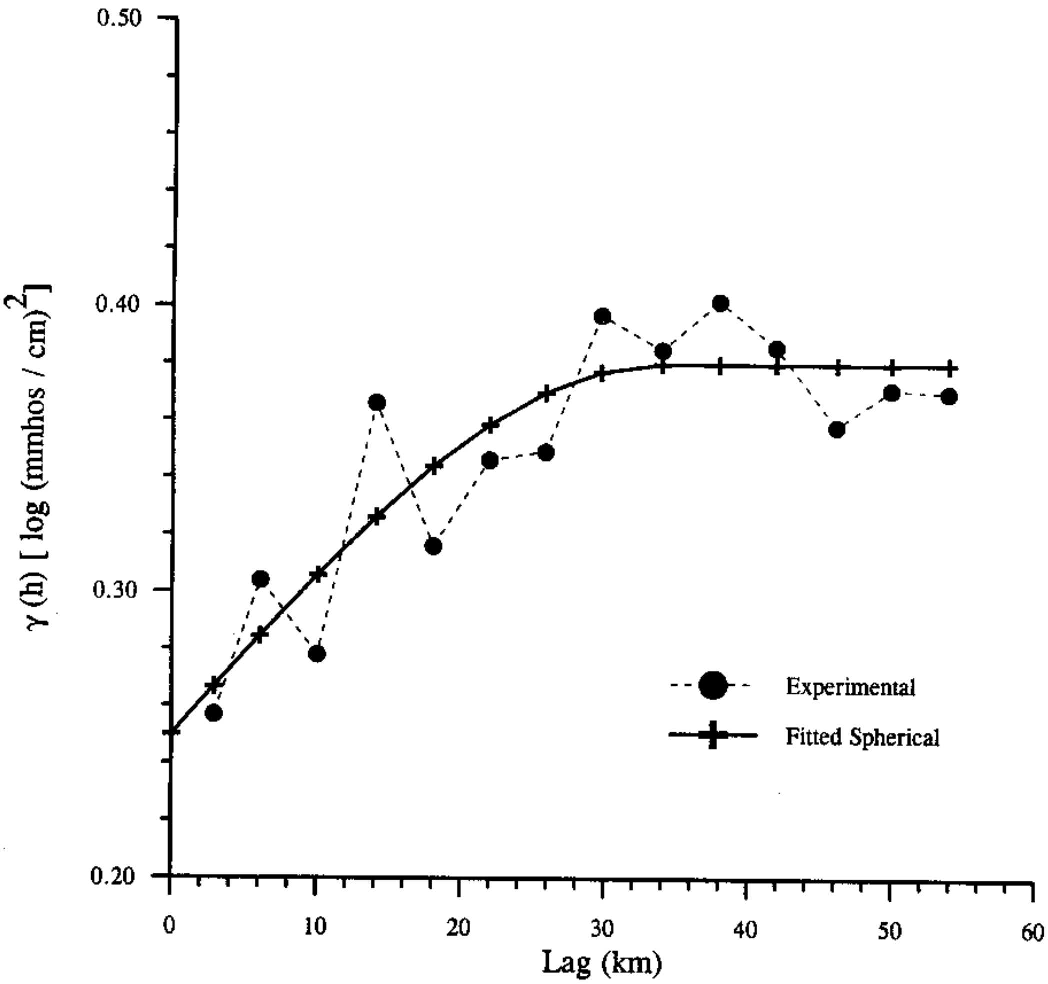


Fig. 17. Semivariogram of logarithm of conductance

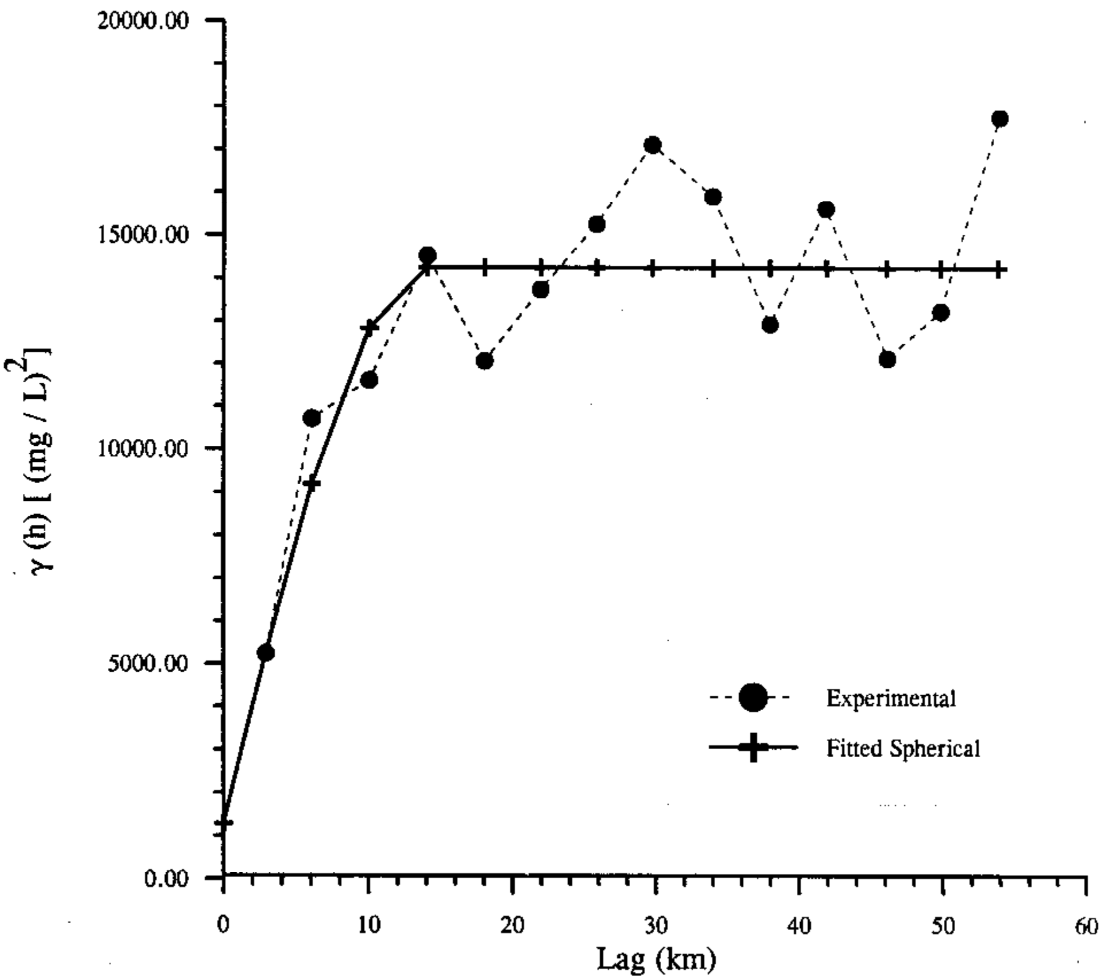


Fig. 18. Semivariogram of chloride concentration

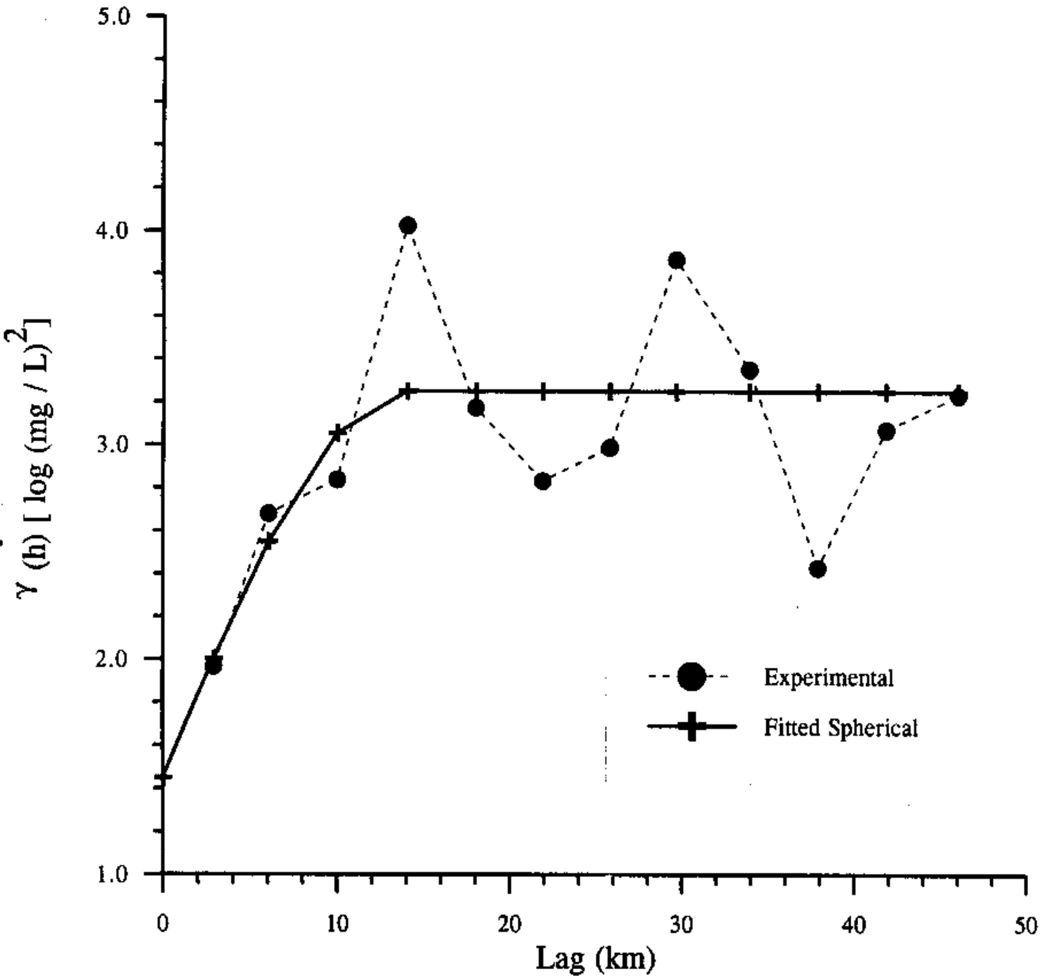


Fig. 19. Semivariogram of logarithm of potassium concentration

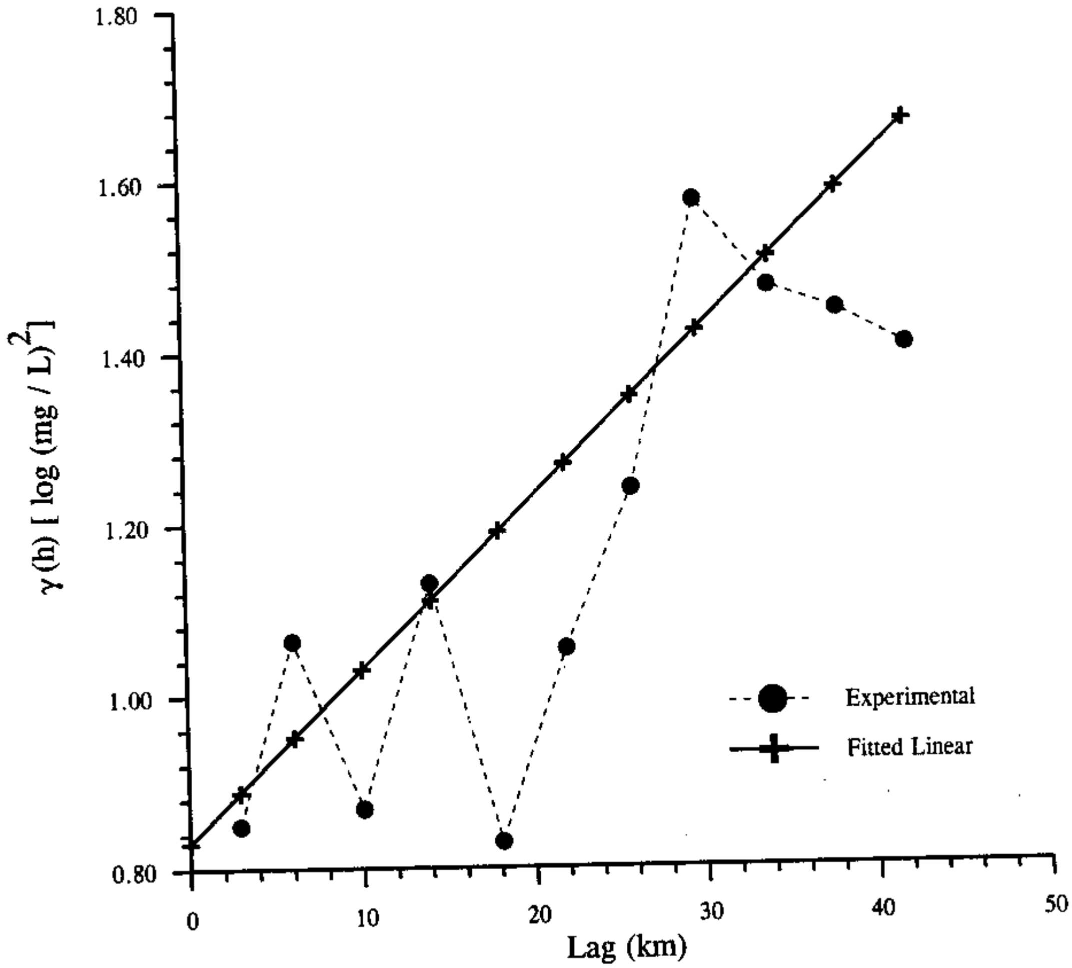


Fig. 20. Semivariogram of logarithm of sodium concentration

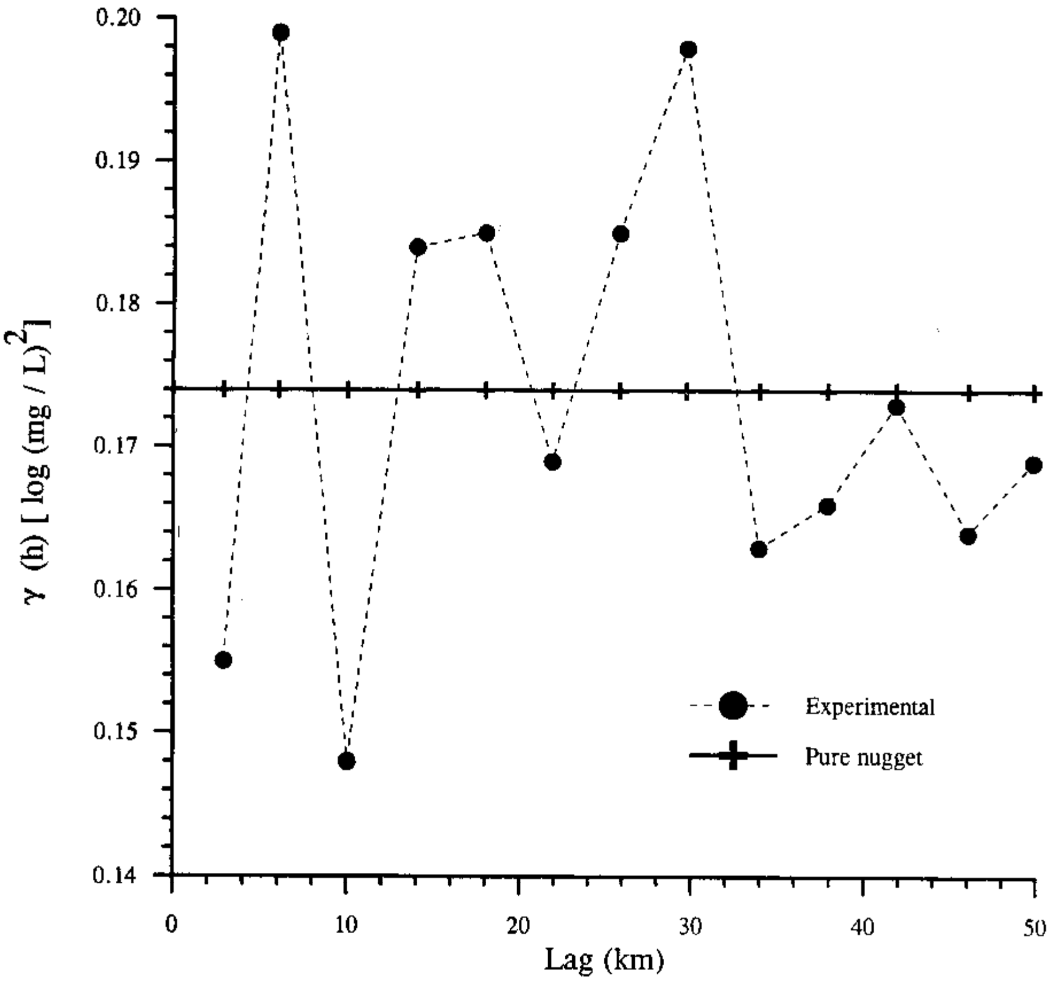


Fig. 21. Semivariogram of logarithm of alkalanity

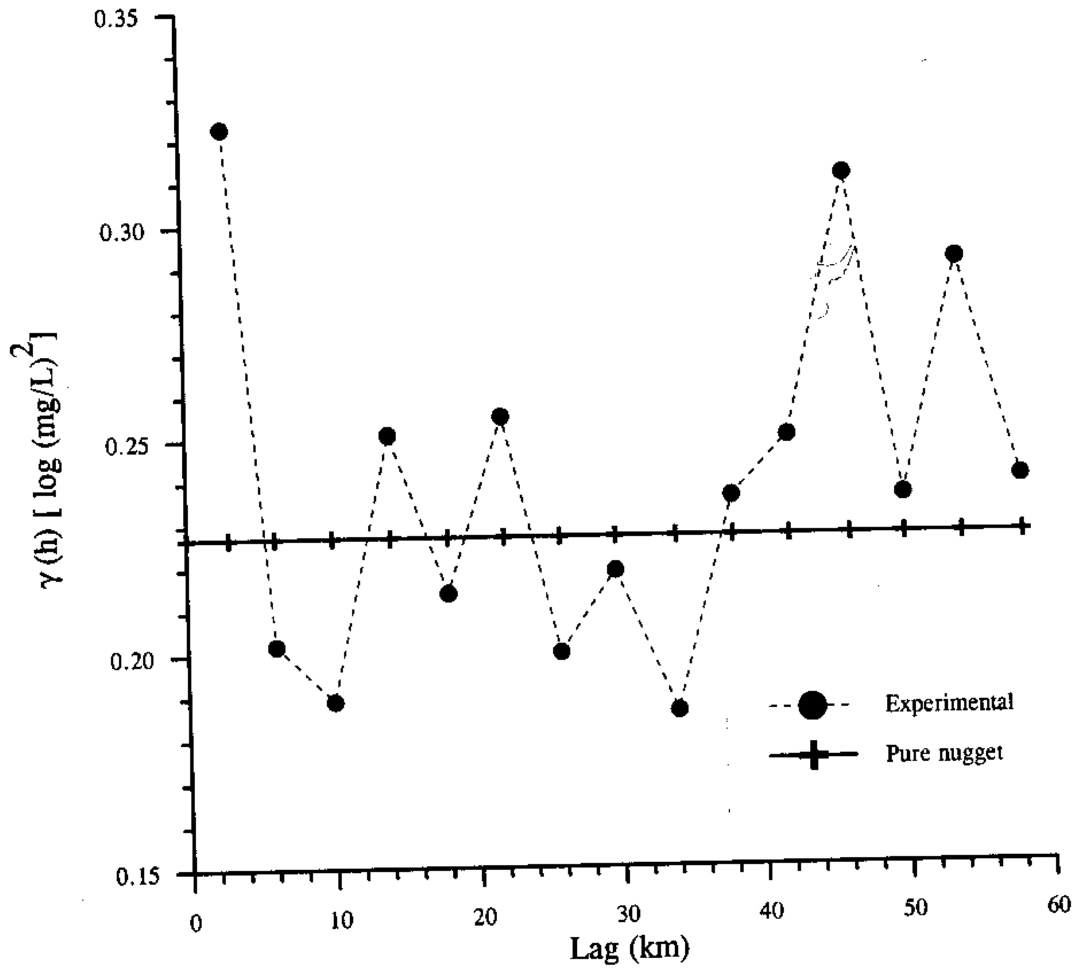


Fig. 22. Semivariogram of logarithm of calcium concentration

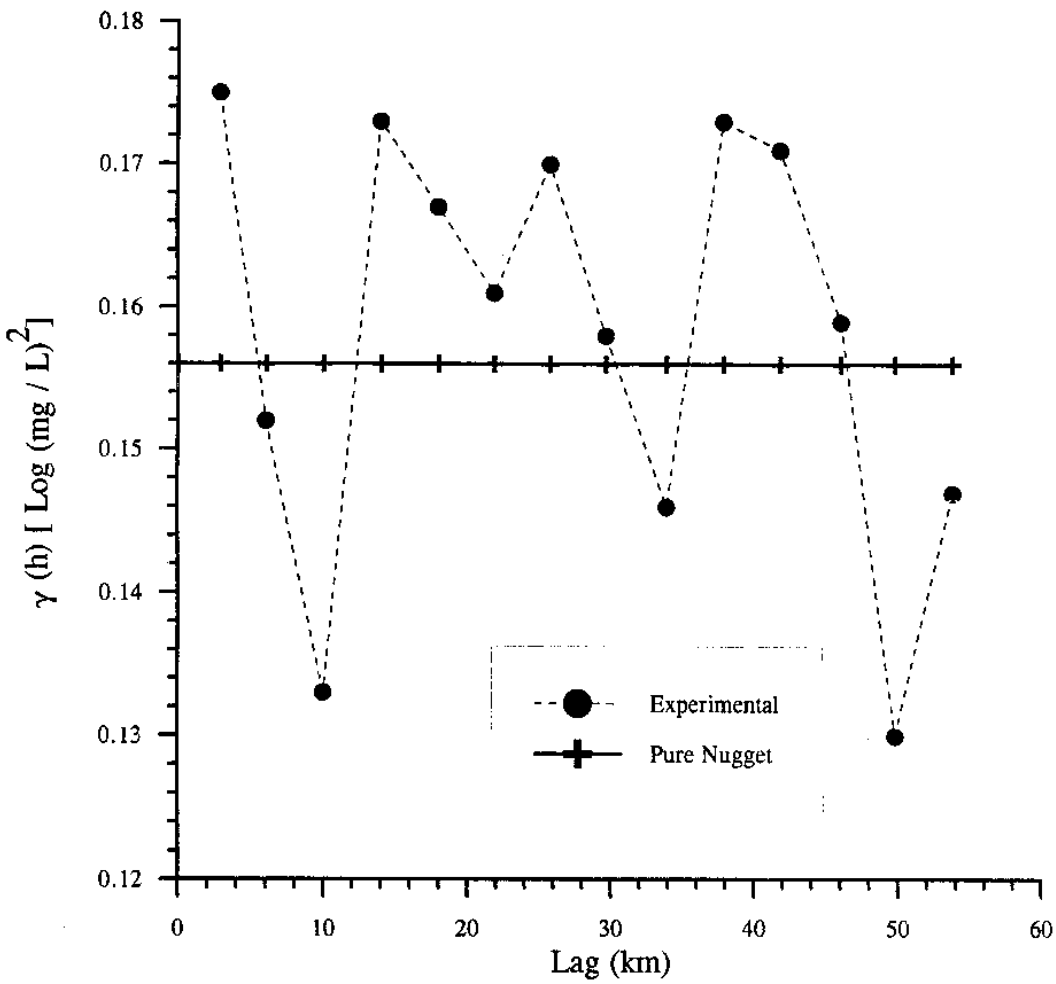


Fig. 23. Semivariogram of logarithm of total hardness

The experimental semivariogram for pH of groundwater is shown in Fig.15. The shape of the experimental semivariogram show a clear spatial structure (spatial dependence), with the semivariogram exhibiting nugget, sill and range. Visual examination of the semivariogram shows that the semivariance is reaching a constant value at a distance of about 10km. The point at which semivariogram remained constant or leveled out is taken as the point where the range of dependency ends. Structural analysis is carried out on this experimental semivariogram. The shape of the semivariogram, shown in Fig. 15, indicates that spherical, exponential, and gaussian model can be fitted to it. Weighted least square error method was used to fit these models. It was found that the spherical model having a nugget of 0.012, sill of 0.082 and a range of 10km best fit the semivariogram. The result of all other fits are shown in Table 3. A range of 10 km indicates that pH of groundwater is spatially related upto a distance of 10km. The fitted spherical semivariogram is also shown in Fig. 15.

Fig.16 shows the semivariogram for the magnesium concentration in the groundwater. The shape of the experimental semivariogram is similiar to that of pH. In this case, a spherical model having a nugget of 168.0, sill of 524.0 and a range of 14km best fit the semivariogram. So, the magnesium concentration in the groundwater are spatially related upto a distance of 14km. Similarly Fig. 17, 18 and 19 shows the semivariograms of the conductance, chloride and potassium in the groundwater. Semivariogram shape for all the three parameters is similar to the earlier one. For conductane a spherical model with a nugget of 0.25, sill of 0.38 and a range of 34km, for chloride a spherical model with a nugget of 1285, sill of 14282 and a range of 14km and for potassium a spherical model with a nugget of 1.45, sill of 3.25 and a range of 14km best fits the experimental semivariogram. The above ranges indicate that conductance of groundwater is spatially related upto a distance of 34km whereas magnesium and chloride are spatially related upto a distance of 14km. Fig 17 to 19 also shows the fitted spherical model for the three. The above spatial structure analysis of the groundwater quality parameters indicate that the distance between observation points should be at least equal to the range of the variable to get the value of respective variables that are spatially independent.

Table 3 Details of model fits to various ground water quality parameters

Sr No	Parameter	Spherical Model				Gaussian Model				Exponential Model				Linear Model		
		C_0	C	a	e	C_0	C	a	e	C_0	C	a	e	C_0	b	e
1	pH	0.012	0.07	10.0	0.010	0.026	0.056	5.4	0.012	0.00	0.11	3.0	0.033	0.05	0.00	0.017
2	Na	0.59	0.54	8.6	0.26	0.89	0.87	36.2	0.19	---	----	---	---	0.83	0.02	0.15
3	K	1.45	1.8	14.0	0.43	1.69	1.6	6.5	0.54	---	---	---	---	2.26	0.03	0.62
4	EC	0.25	0.13	34.0	0.021	0.26	0.11	15.2	0.025	0.23	0.15	13.8	0.023	0.27	0.00	0.033
5	Cl	1285.9	12997.4	13.3	1805.1	3031.2	11099.5	6.0	1829.1	0.0	17894.0	4.4	3805.3	7207.9	222.2	3020
6	mg	168.0	356.0	14.0	84.0	233.8	294.2	7.5	84.6	23.5	503.9	4.4	85.3	309.5	7.8	117.0

C_0 = Nugget effect　　C =Sill - Nugget effect　　a = Range

e = standard error　　b = slope at the origin

Fig.20 shows the experimental semivariogram for the sodium concentration in groundwater. The shape of the experimental semivariogram indicates that the semivariance is not reaching a constant value and its value goes on increasing. A linear model with a nugget of 0.83 and a slope of 0.02 best fits the experimental semivariogram. This shape of semivariogram indicates that there is no defined range with constant semivariance.

The experimental semivariogram of alkalinity (Fig.21) lacks spatial structure and shows no range of dependency. Most semivariogram values scatter around the variance for all lags. Alkalinity had a large point to point variation at short distances of separation and an absence of spatial correlation at the scale used. Thus the semivariogram for alkalinity did not have any structure. There was no increase to maximum value. A pure nugget effect was observed, indicating short range variability in alkalinity. Semivariogram for the calicum (Fig.22) and total hardness (Fig.23) in the groundwater is similiar for that of alkalinity i.e. pure nugget effect. The nugget effect value is equal to the variance of the respective observed parameter. The lack of defined structure suggets that a random sampling scheme of the groundwater could provide the information in regard to these three groundwater quality variables.

5.2 Kriging

For interpolated estimation of groundwater quality variables at any unsampled location, kriging has been applied. The study area was divided into a square grid of 5km and groundwater quality was estimated at each of the grid nodes using the finally selected model and the observed data for the respective variable. In case of variables that showed a pure nugget effect, the values are interpolated at the nodes of 5km square grid using the inverse square method of interpolation. These estimated quality values were used with the SURFER software to draw the contour maps. The contour map of the groundwater quality levels so obtained are shown in Fig. 24 to 32.

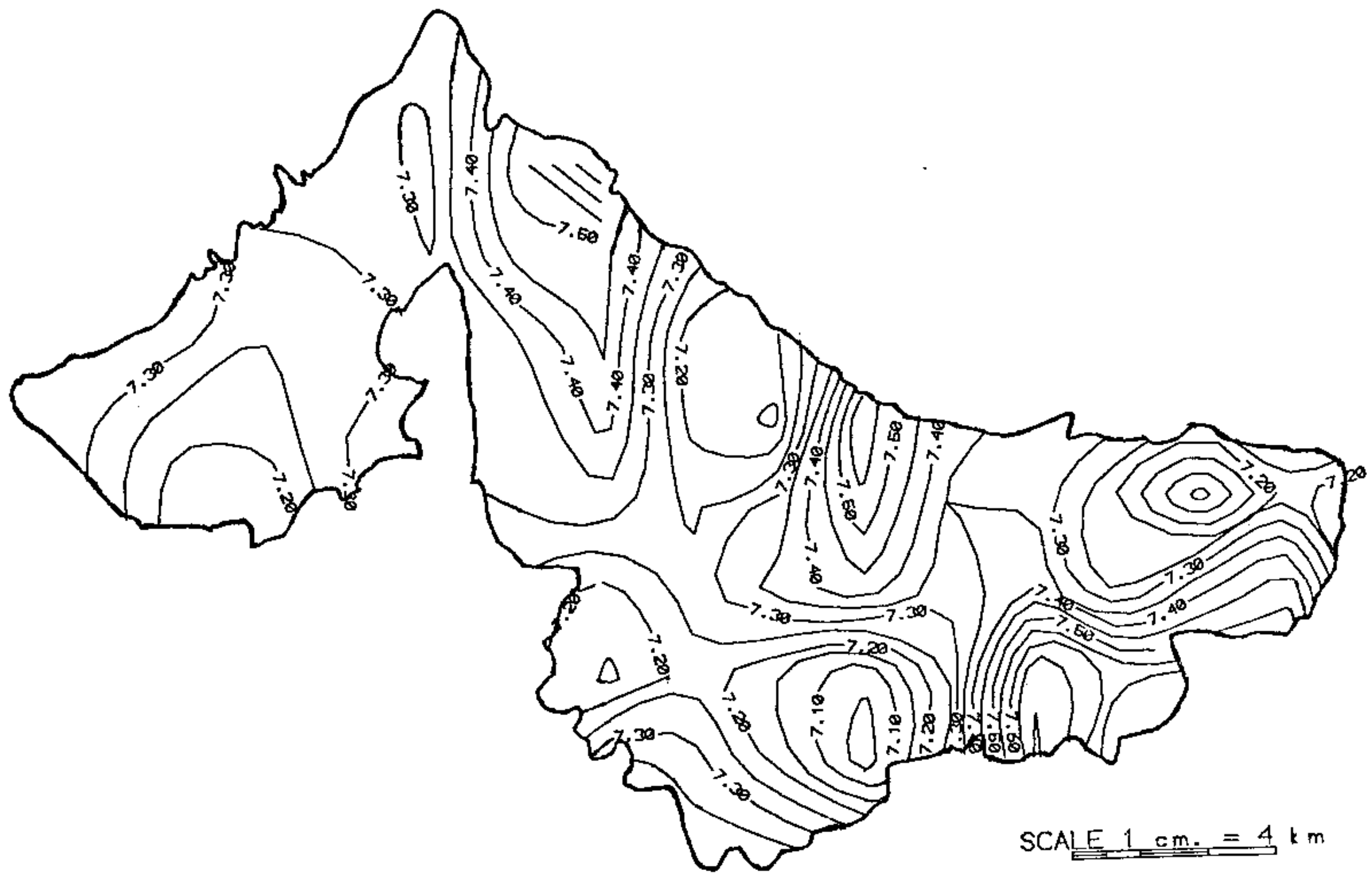


Fig.24. Kriged contour map of pH

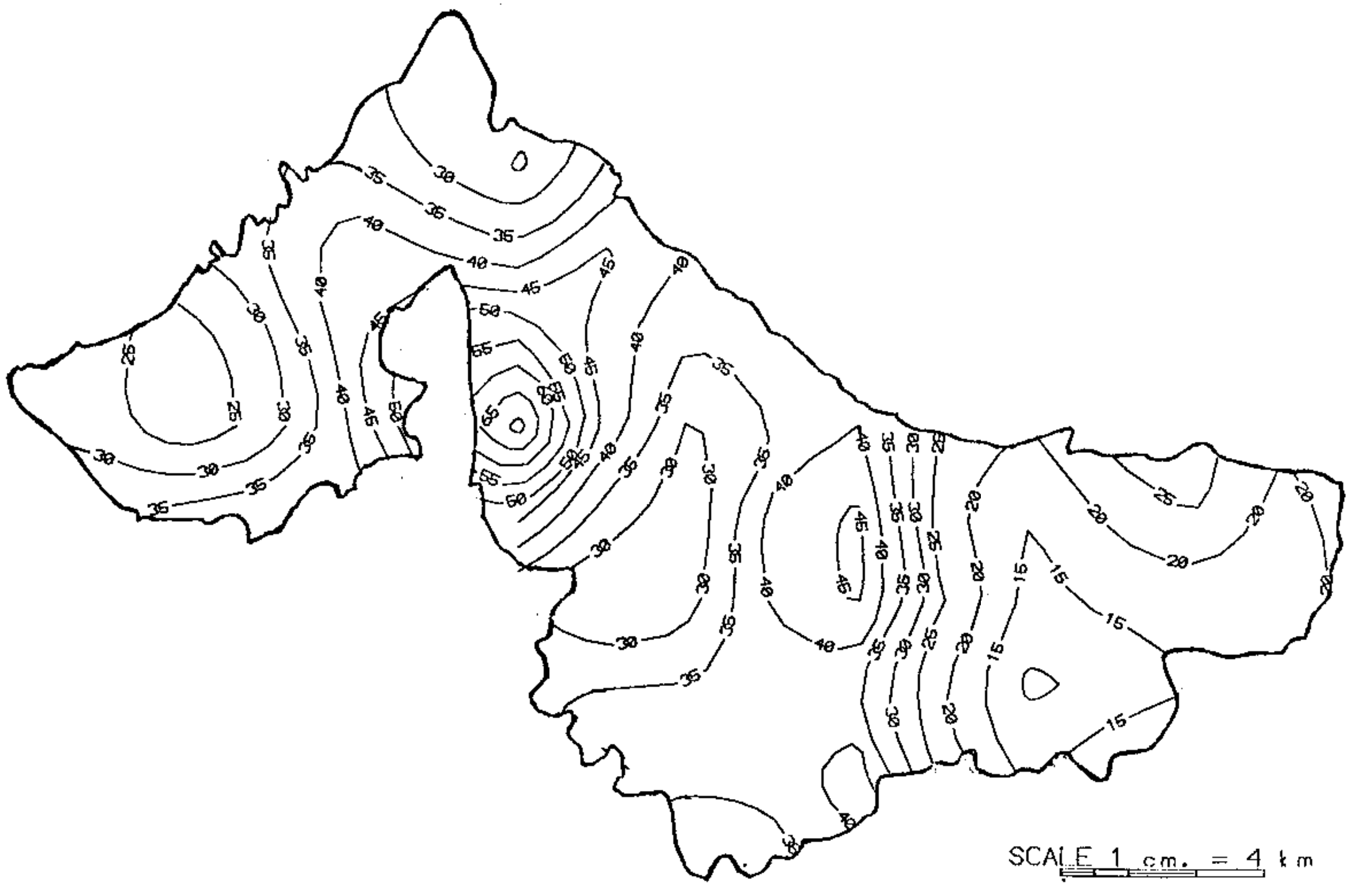


Fig.25. Kriged contour map of magnesium concentration (mg/L)

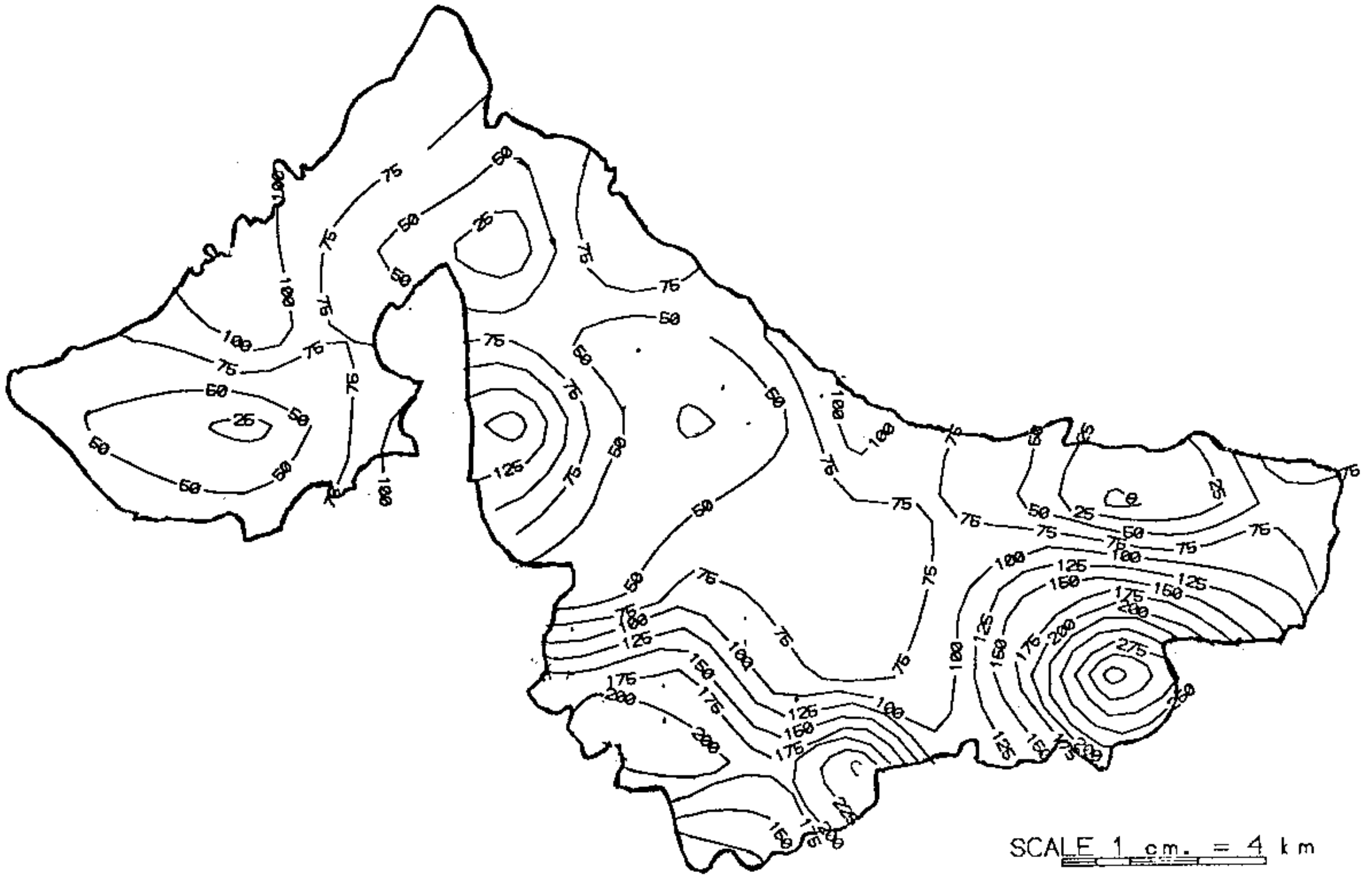


Fig.26. Kriged contour map of chloride concentration (mg/L)

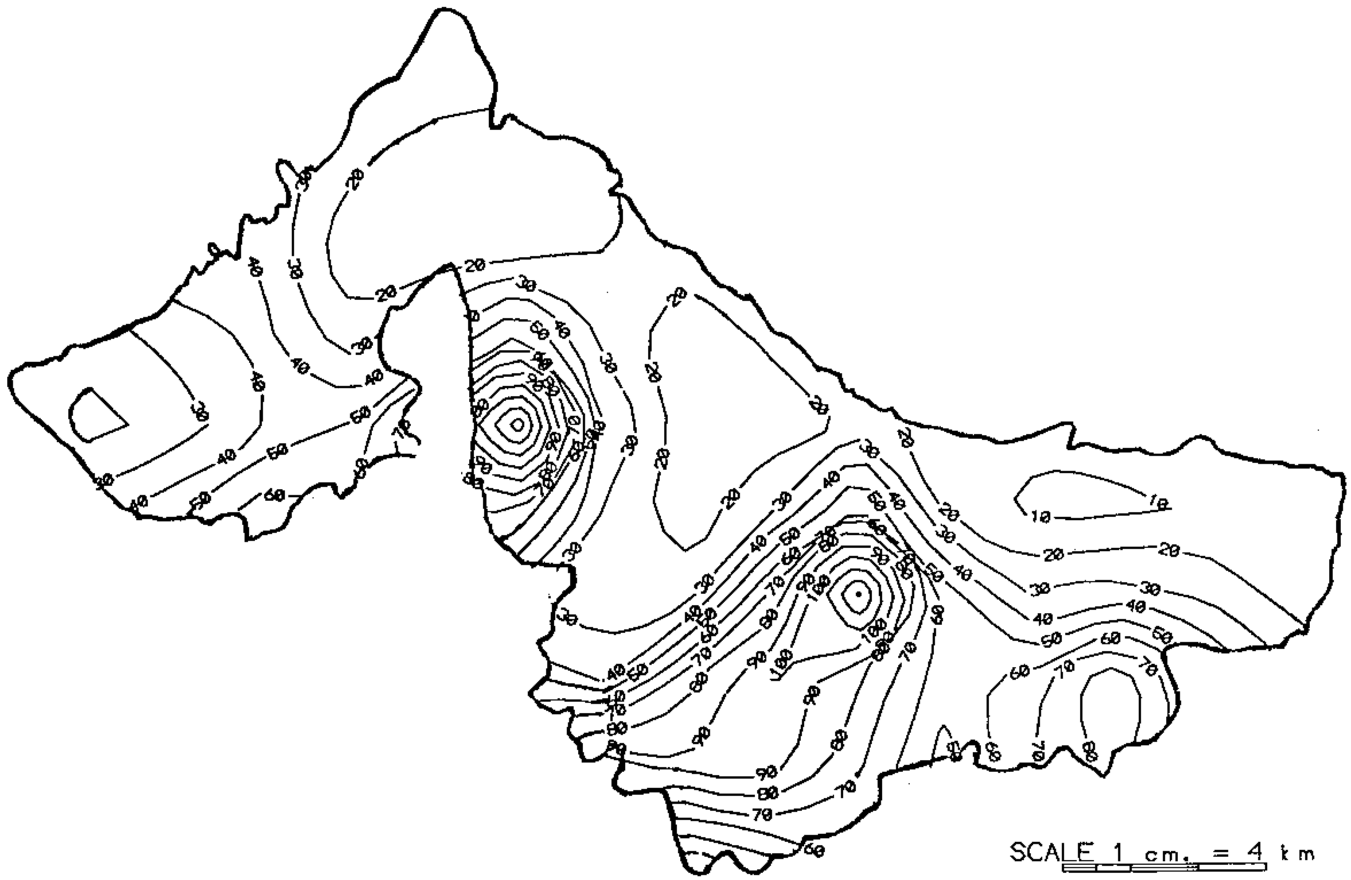


Fig.27. Kriged contour map of potassium concentration (mg/L)

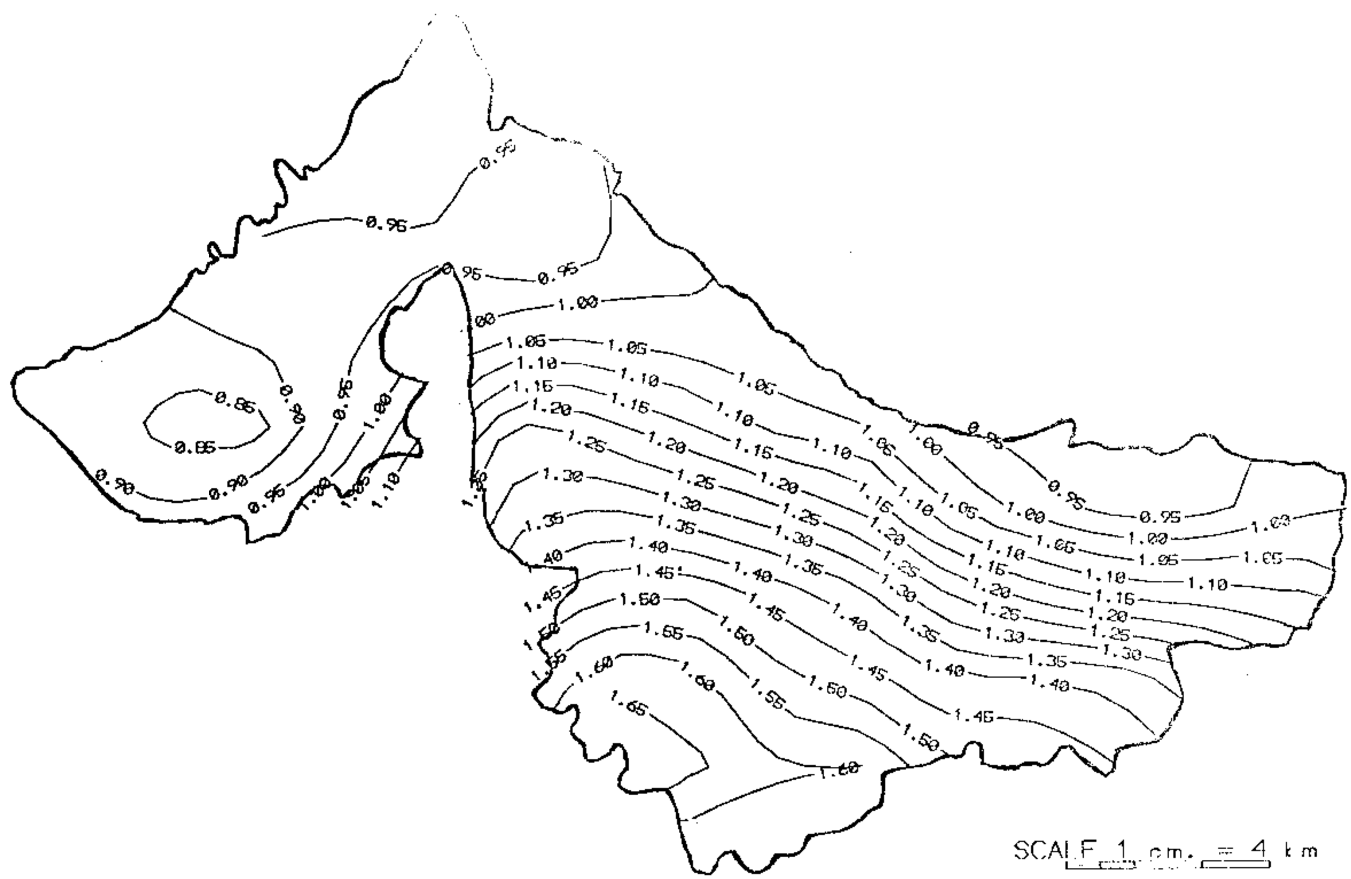


Fig.28. Krigod contour map of conductance (mmhos/cm)

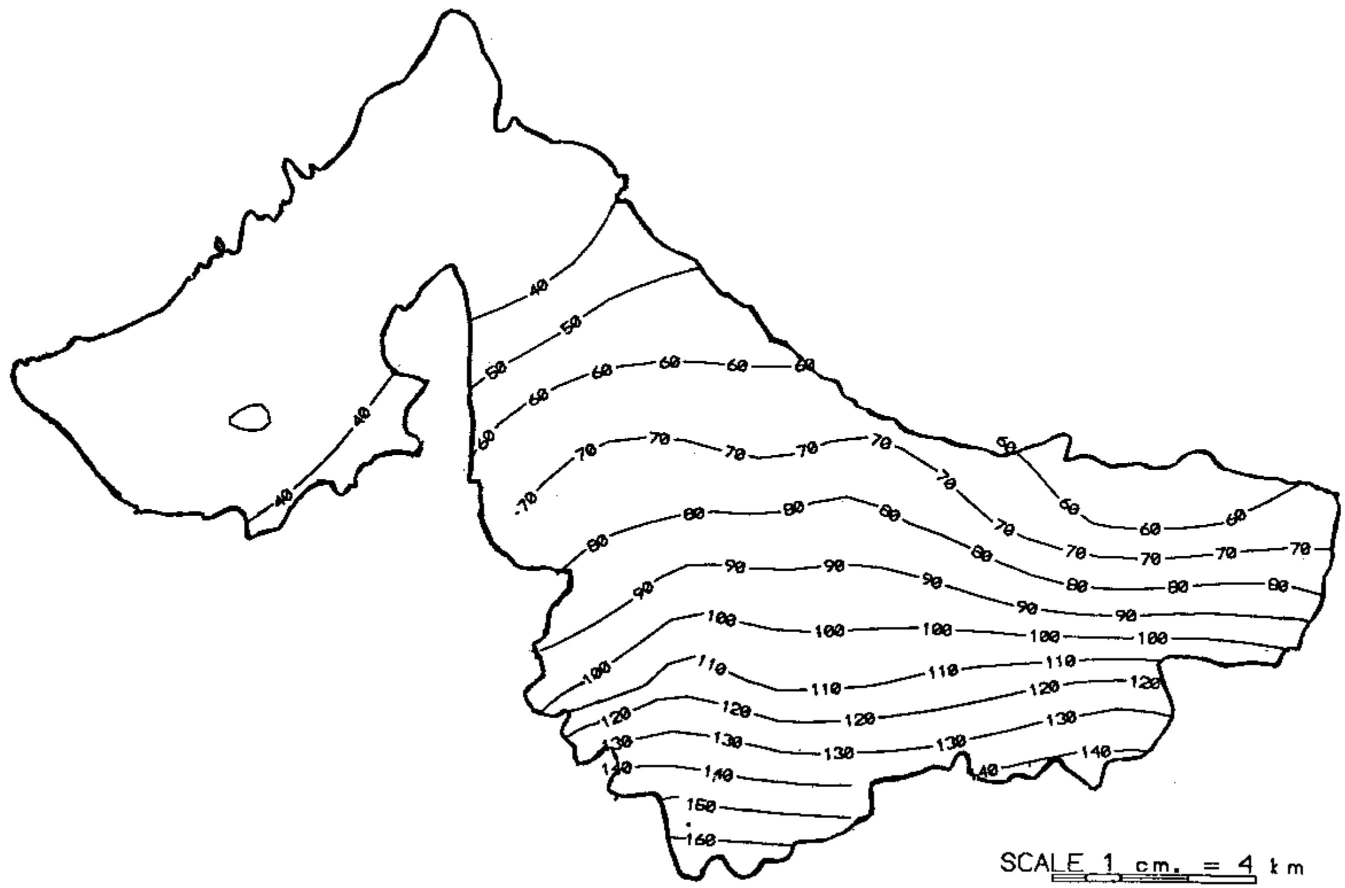


Fig.29. Kriged contour map of sodium concentration (mg/L)

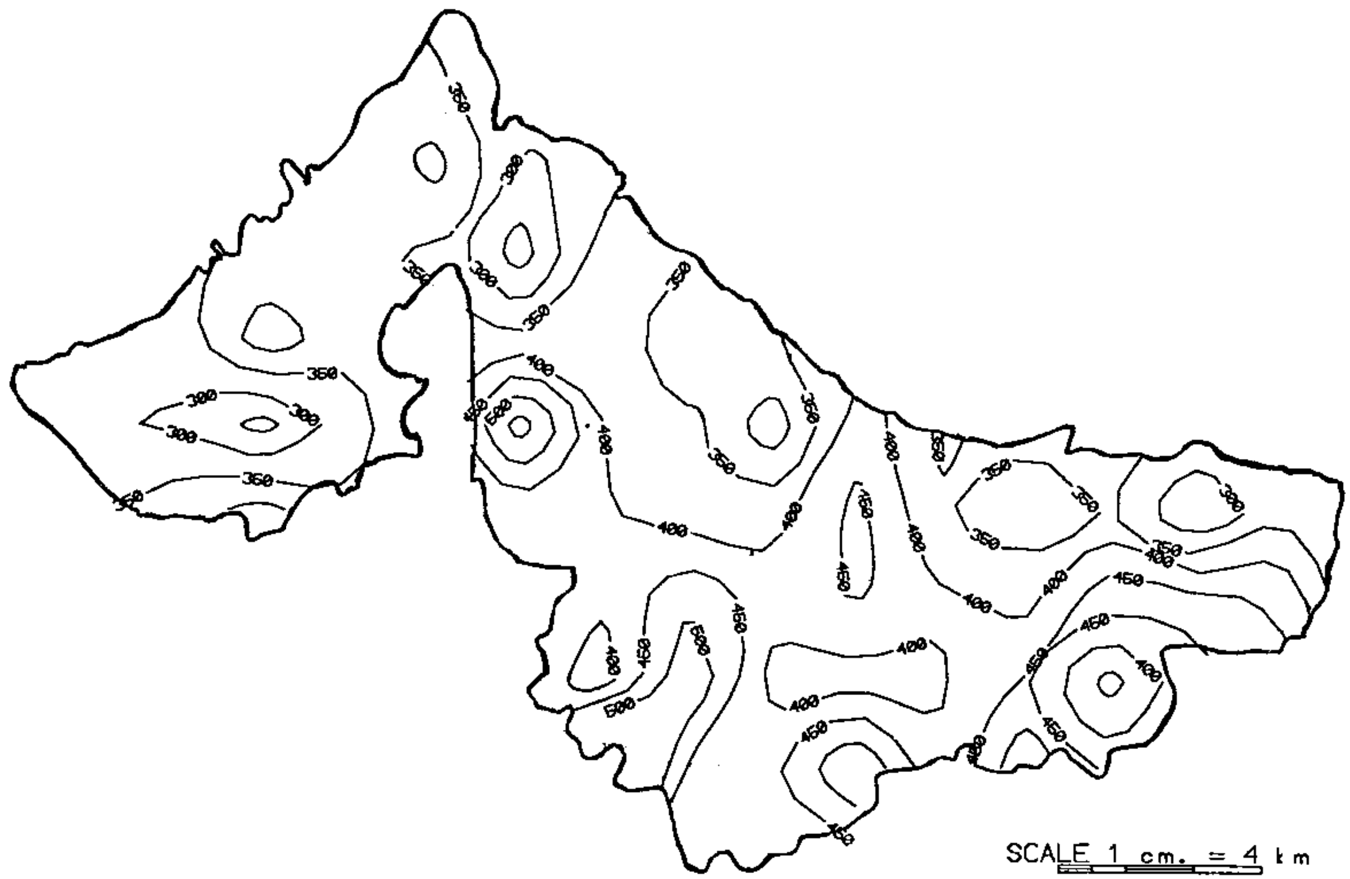


Fig.30. Contour map of alkalinity (mg/L)

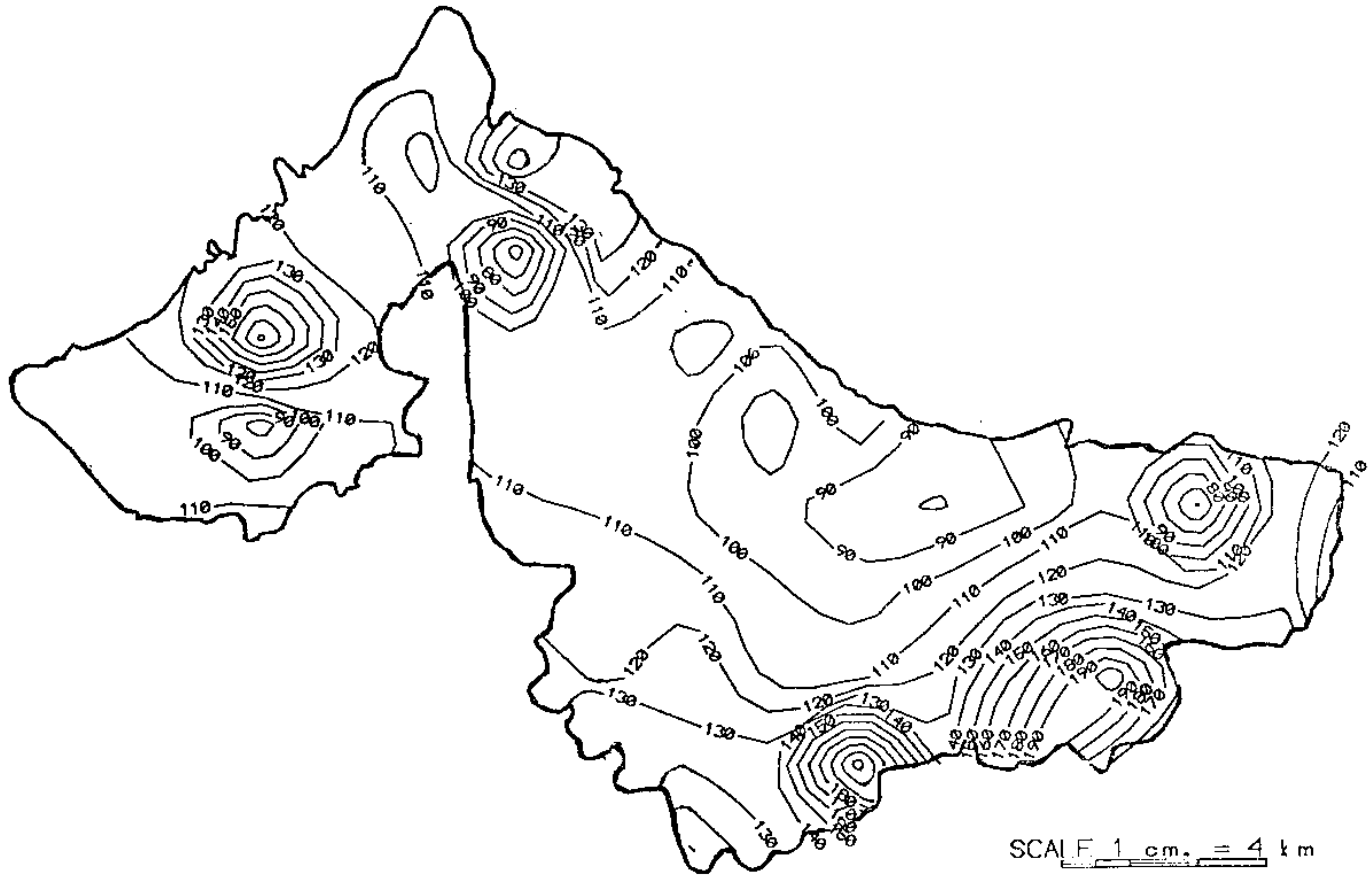


Fig.31. Contour map of calcium concentration (mg/l)

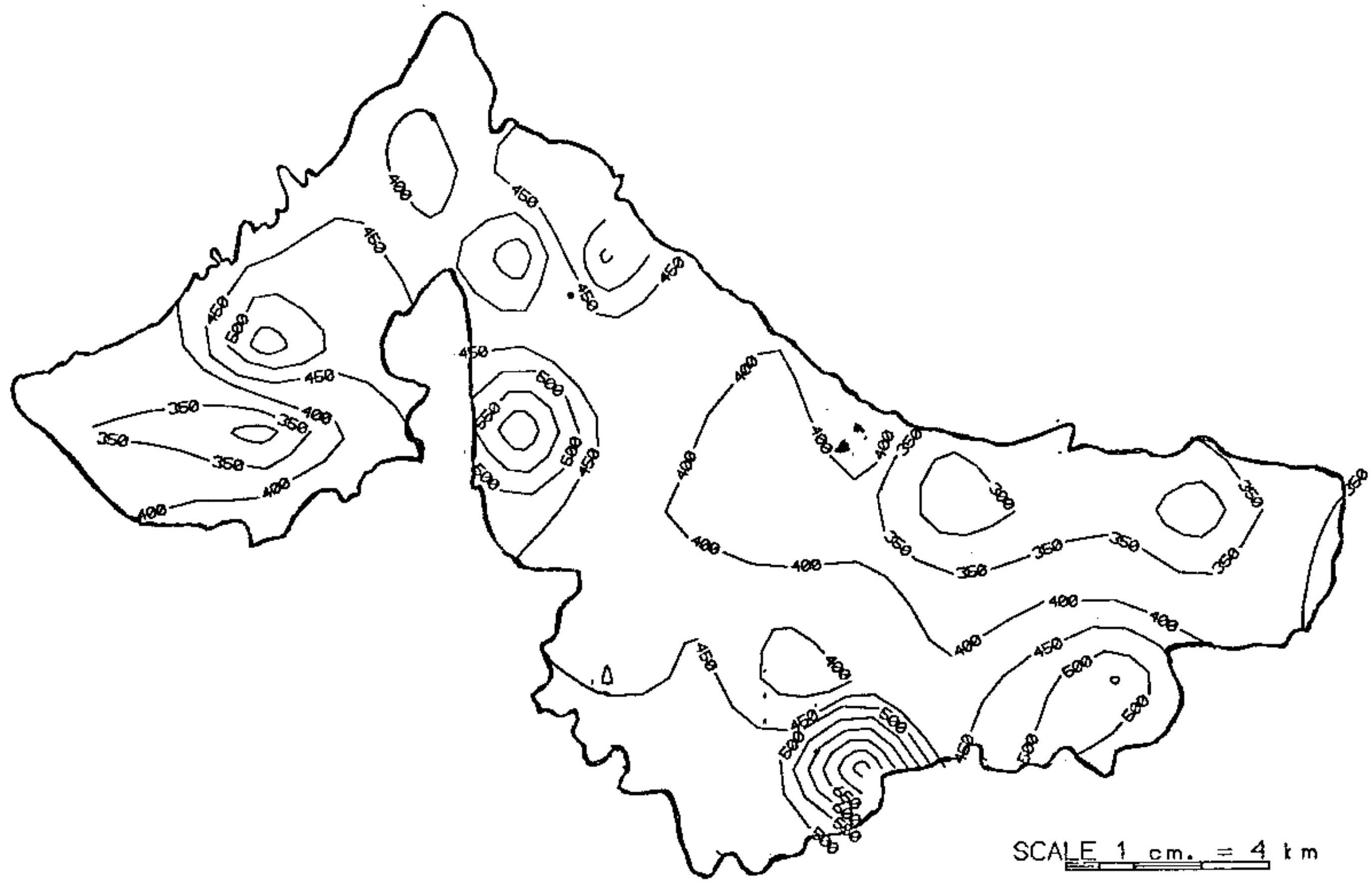


Fig.32. Contour map of total hardness (mg/L)

The kriged contour map of pH (Fig.24) shows high spatial variablity of pH of groundwater. There are many 'hills' (the location having high value) and 'valleys' (the location having low value) at close distance in the contour map. Also the contours are spaced very closely. The kriged contour map of magnesium concentration (Fig.25) also shows high spatial variation. The distance between one 'hill' to adjacent 'valley' or vise versa is more as compared to that of pH. Note that the spatial range of magnesium concentration (14km) is more as compared to that of pH (i.e. 10km). The contour map of chloride concentration (Fig.26) and potassium concentration (Fig.27) are similiar to that of magnesium. These three groundwater quality parameters have the same spatial range (i.e. 14km). The kriged contour map (Fig.28) of conductance of groundwater (having a spatial range of 34km) shows the presence of only few 'hills' and 'valleys'. In the kriged contour map (Fig.29) of sodium concentration (having unlimited range) there is no 'hill' or 'valley'.

Fig.30, 31 and 32 shows the contour map of alkalinity, calcium concentration and total hardness of groundwater respectivily. As pure nugget effect model has been fitted to the semivariograms of these parameters, the contour maps are prepared using the inverse square distance method. All the three contour maps are similiar to each other and have high variability from one location to another. Comparing the kriged contour maps (Fig.24 to 29) with these three contour maps (Fig. 30 to 32) indicate that in the kriged contour maps, the contours are smoother than the contours in these maps.

6.0 CONCLUSIONS

In this study, geostatistical techniques is applied to the groundwater quality data of December 1994 in parts of Jammu district of J&K state. Semivariogram analysis indicates that the pH, magnesium, potassium, conductance, sodium, and chloride are spatially auto-correlated, whereas alkalinity, calcium and total hardness are found to be spatially non-autocorrelated. Spherical model is found to be best model representing the spatial variability of pH, magnesium, chloride, potassium and conductance, whereas, linear model for sodium. pH is found to be spatially related upto a distance of 10km, magnesium, chloride and potassium upto a distance of 14km, and conductance upto a distance of 34km. This distance represents the minimum distance, on an average, at which maximum variation occurs. The data suggests that the distance between observation points for the measurement of pH of groundwater should be at least 10km to get pH values that are spatially independent. An important application of this investigation is that it gives guidelines for the selection of observation points to study the groundwater quality. The results of this study emphasize that groundwater quality commonly have spatial dependence and that understanding such structure may provide new insights into groundwater quality behaviour over the aquifer.

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