AN ILLUSTRATED COLLECTION OF GROUNDWATER PROBLEMS

prepared by

Sunderrajan Krishnan

as part of

Groundwater Governance in Asia: Theory and Practice

Contents

About the front page

Wells represent the interaction between aquifers and society. They are a reflection of geology, culture and local engineering. Here some collection of pictures taken from across South Asia are produced. These pictures are from:

Basaltic, Alluvial, Sandy, Crystalline and Hilly aquifers of Nepal, and from Gujarat, Madhya Pradesh, Uttar Pradesh and Coimbatore in India.

Nomenclature

H: Hydraulic head
i: Hydraulic gradient
K : Hydraulic Conductivity
K_{eff}: Effective Conductivity
v: Darcy velocity
v_g: Velocity of groundwater flow
q: Porosity
S_Y: Specific Yield
S: Storage Coefficient
ΔS: Change in Storage

1. Introduction

The purpose of this collection is to guide a beginner to groundwater hydrology through the basic concepts in this subject. The problems begin with fundamentals of the subject and finally some problems which test the comprehensiveness of understanding are developed. Most problems are illustrated and a real-world situation is related with the problem.

The collection is part numerical-based, part software based. Computer tools have the power of visualization and extrapolation beyond the current problem. The purpose of computer-based exercises are therefore to go beyond the problem developed here and use them to gain further understanding of concepts.

Most of the numerical problems have been adapted from exercises provided in textbooks. The references to the original problems have been provided along with the text here. In this version, further questions are added to some problems along with graphics and in some cases with change in numerical quantities. Any deviation from the original purpose of these problems developed by the original authors is sincerely apologized and comments would be welcome on that.

Solutions are provided to all problems in this collection. The earnest student is requested to apply utmost effort before having a look at the solutions. After all, these problems have a self-learning purpose and merely reading the solutions will not serve the purpose of consolidating the theory.

One should use this problem set in conjunction with a book on groundwater hydrology. Here we have provided the basic formulae and concepts that are necessary to solve the problems here. One should refer to textbooks to know more about the assumptions behind these formulae and situations in which they can be used.

Above all, one should treat a journey through these problems as a beginner's training. For further expansion into this fascinating subject, there is no replacement of a combination of field studies and relating them to the theory. In that context, a knowledge of the numerical aspect of this subject and application of theory will enhance understanding and deepen interest in groundwater studies.

2. Numerical problems

i) Water balance

The hydrologic cycle is a continuous movement of water through the earth system at various time and space scales. Within this cycle, any smaller system such as a river basin, an aquifer or a plant can be seen as a conduit for water flow and storage. Unless water is created within any of the components of this system, the change in storage should be reflected by water flowing in and out.

$$\Delta S = F_{out} - F_{in}$$ **Equation 1**

i.e the difference between flow out F_{out} and flow in F_{in} is the change in storage ΔS. This same concept can be applied at any time or space scale to derive various types of water balance equations. One such example for groundwater is:

$$\Delta S = F_{out} + P + E - F_{in} - R_e - R_p$$ **Equation 2**

where the change in storage in an aquifer ΔS is given by inputs in terms of flow inwards, recharge from precipitation R_p, and from return flow R_e and outputs in terms of flow outwards laterally and vertically, Pumping P and Evapotranspiration E.

In every specific example, we come across special requirements for modifying the water balance and one needs to tailor to the case keeping in mind the basic concept of mass balance.

Problem 1 Yearly groundwater balance over an aquifer

(Domenico and Schwartz, 1990)

Availability of electricity and subsidy for irrigation wells and pumps led to a sudden rise in pumping in Indian Punjab. A research institution took up the task of analyzing the situation for one aquifer in this state and the first step was to compute the annual groundwater balance for the past few years. From their calculation using field data and secondary information, they came up with the following table:

Table 1: Groundwater Balance over an Aquifer

Time Year	Recharge from Direct Precipitation m^3	Net Recharge from Stream flow m^3	Discharge by Pumping m^3	Discharge by natural Evapotranspiration m^3	Change in Ground Water Storage m^3
1	3×10^7	0	0	3×10^7	?
2	3×10^7	6×10^5	1×10^7	3×10^7	?
3	3×10^7	1×10^6	3×10^7	9×10^6	?
4	2.8×10^7	2×10^6	3.5×10^7	5×10^6	?
5	2.5×10^7	3×10^6	3.5×10^7	3×10^6	?
6	3.5×10^7	4×10^6	4×10^7	1×10^6	?
7	3.5×10^7	4×10^6	4.2×10^7	1×10^6	?
8	3.5×10^7	4×10^6	4×10^7	1×10^6	?

a) Compute the changes in groundwater storage. Is the aquifer being "overexploited"? How would you qualify this statement?
b) How much proportion of natural recharge is available for natural ET? How is this proportion changing over the years? Why?
c) If this aquifer stores 1 x 10^6 m^3 of water for every meter of depth, compute the changes in water levels. What is the trend in water levels over the years?
d) Towards the end of Year 8, approximately how much would pumping need to be limited to so that there is no further drop in water levels?

Problem 2 Daily Soil moisture balance for a single storm

(give source)

Often, we need detailed monitoring of individual storms so that the excess soil moisture and recharge happening from such storms can be estimated. In such a case, atleast daily monitoring is needed. Jaisalmer lies in the Thar desert region of India. The rainfall here is highly variable and recharge to groundwater is minimal. Much of water evaporates and low amount of excess soil moisture is available. Data from a single storm in western arid part of India, Jaisalmer, for September 1976 is shown in the table. The initial soil moisture is 100.1 mm.

a) Compute the final soil moisture after the storm. Use this soil moisture balance equation:

$$\text{Soil mois}_{curr} = \text{Soil mois}_{prev} + \text{Rainfall}_{curr} - \text{ET}_{prev} - \text{Moisture surplus}_{prev}$$

where curr and prev denote the current and previous day. Apply this equation for each day successively to compute the final soil moisture.

Day	Rainfall	Soil Moisture	Actual ET	Moisture surplus
1	mm	mm	mm	mm
2	0	100.1	4.3	0
3	45	?	6	0
4	26	?	6.4	4.4
5	19	?	6.4	12.6
6	19	?	6.4	12.6
7	26	?	6.4	19.6
8	0	?	6.4	0

b) What is the total soil moisture surplus? If 20% of this soil moisture surplus infiltrates as recharge to the groundwater, what is the rainfall-infiltration factor for this storm as a whole?

Table 2: Daily soil Moisture during a single storm

ii) Hydraulic Head

The total head of groundwater at a given location is the sum of elevation head, pressure head and velocity head (generally considered negligible). The total head above a datum is give by:

$$H = H_P + H_E \qquad \textbf{Equation 3}$$

The elevation head H_E is measured from a fixed datum, such as mean sea-level. The pressure head H_P could arise from different means such as pressure of from ponding or column of water above.

Hydraulic gradient is any given direction is the gradient of the hydraulic head. It is denoted as i.

Problem 3 Measuring elevation and pressure head

(Domenico and Schwartz, 1990)

Deep investigations of a confined aquifer revealed that the structure of the aquifer is in the form of a folded bed. The weight of the overload material on this syncline is such that the overburden pressure of material is much higher at the lower parts than at the higher parts leading to higher pore pressure at the lower parts of the aquifer. This was suspected to lead to groundwater flow against the topographic gradient. In order to test this, a set of piezometers were installed in the aquifer taking care to see that all of them reach this same confined aquifer.

In the accompanying diagram assume a hydraulic gradient of 100 ft/mile, with the water level at point A being at an elevation of 900 feet above sea level. Assume further that the water level in all the piezometers is at the top of the piezometers. The piezometers are located 1 mile apart. Calculate the following blanks:

1

	A	B	C	D
Depth of Piezo(ft)	600	575	150	25
Total head(ft)	900	?	?	?
Pressure head (ft)	?	?	?	?
Elevation head (ft)	?	?	?	?

Table 3: Head at different locations

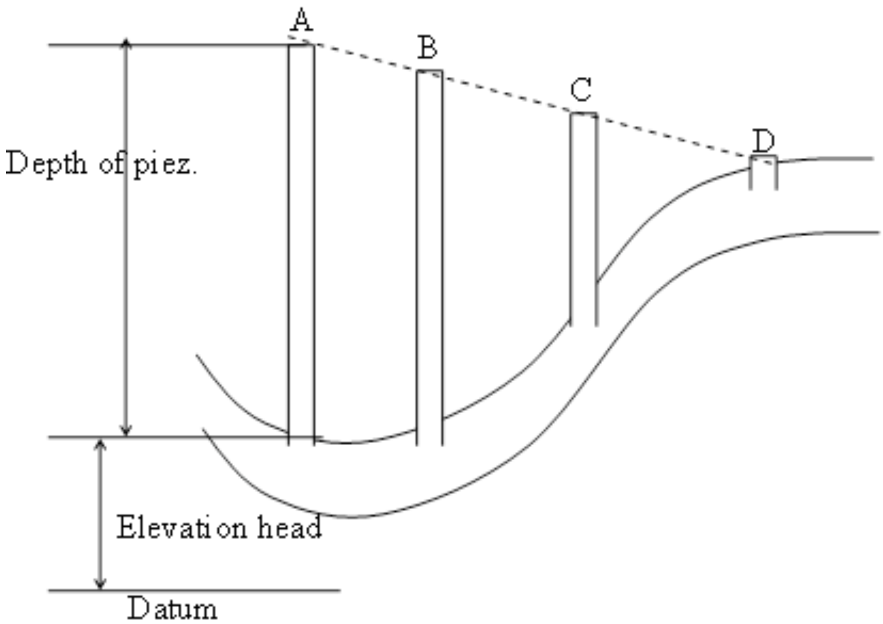


Figure 1: Elevation and Pressure heads at different piezometers

iii) Darcy's Law

As observed in experiment by Darcy, the velocity v of bulk of water moving across a cross-section with hydraulic conductivity K and gradient i is simply (Todd, 2003),

$$v = -Ki$$ **Equation 4**

with the negative sign denoting that the direction of decreasing gradient i is the same as direction of flow. This basic Darcy's law is the basis of almost all current understanding of groundwater flow systems apart from rapid flow through fractures and interflow

The primary properties of porous formations useful for understanding groundwater are porosity θ and hydraulic conductivity K. Porosity is the proportion of pores present in a soil or rock formation. Hydraulic conductivity is a measure of how fast water can flow through these pores, a measure of the connectedness of the pores. It has units of velocity. One interesting aspect of hydraulic conductivity is that at the same location, it can have different values depending upon the direction of flow. For example, it might be easier to flow horizontally rather than vertically in a certain layered sedimentary formation. This property of directional variation is called Anisotropy

The velocity v derived from Darcy's law is known as the Darcy velocity. The velocity of groundwater flow through the pores of the medium with effective porosity θ is given by,

$$v_g = \frac{v}{\theta}$$ **Equation 5**

Problem 4 Darcy's experiment

(Karanth, 1987)

It was Darcy's original experiment with sand within a tube that led to the formulation of the law that stands under his name. Even today, this experiment remains a technique to measure hydraulic conductivity in the laboratory.

An inclined cylinder of 20 cm dia. is filled with saturated sand, with k = 10 m/day, is provided with two piezometers 50 cm apart. The opening of the two piezometers have elevation heads of 50 and 30 cm from a common datum and corresponding water levels in the piezometers are 32 and 26 cm. Estimate the flow rate through the cylinder.

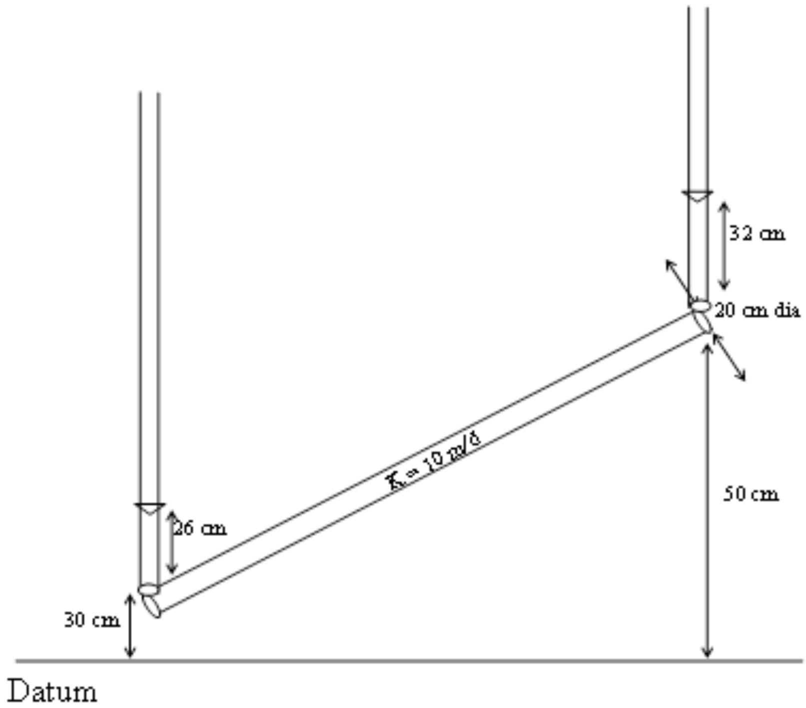


Figure 2: Darcy's experimental setup

Problem 5 Velocity of Groundwater flow

(Karanth, 1987)

Darcy's law provides us with a bulk velocity that can be used for computing discharge through a porous medium. However in actuality, water flows in a tortuous path through pores and velocity fluctuations are observed at different scales. For some cases, detailed velocity profiles might be needed as for transport of solutes. The average of all these velocity fluctuations is the 'velocity of groundwater flow' through the pores of the medium.

Determine the velocity of ground water flow given that:

Average K of aquifer	11.0 m/day
Effective porosity	0.10
Piezometric contour value at up-gradient point	164 m
Piezometric contour value at down-gradient point	152 m
Average distance between contours	18 km

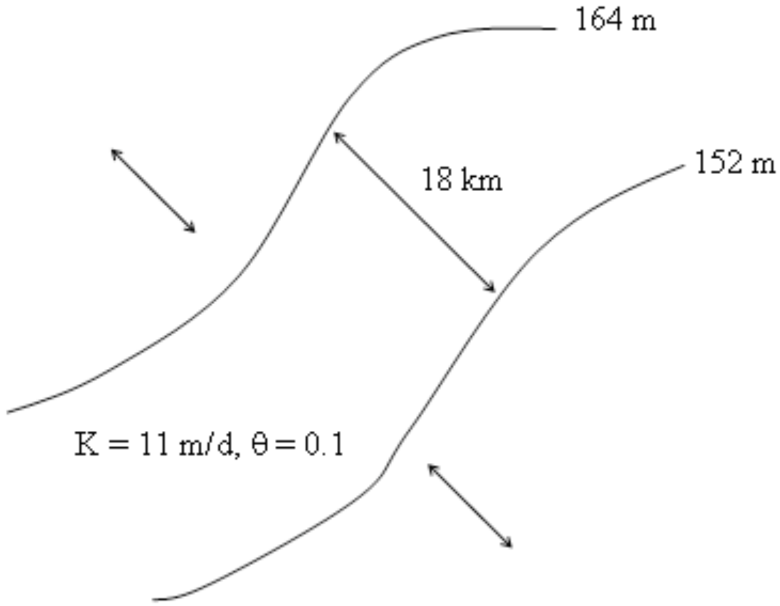


Figure 3: Flow between two head contour lines

Problem 6 Paleo Aquifers and aquifer interaction

(Raghunath, 2002)

In some aquifers, the velocity of groundwater transmission is very slow and water that is recharged several thousand years back is found. Also there is slow interactions between different aquifers. The age of water in different aquifers can be used as an indicator of flow-time between the aquifers.

During hydrogeological investigation two potential aquifers 32 km apart, were located, one being 5,000 years and the other 25,000 years old. They were found to be connected by a water bearing stratum of 30 m thickness running inclined at 20 m/km. From a few observation wells, the hydraulic gradient was found to be 0.2 m/km. Determine the transmissibility of the water bearing stratum.

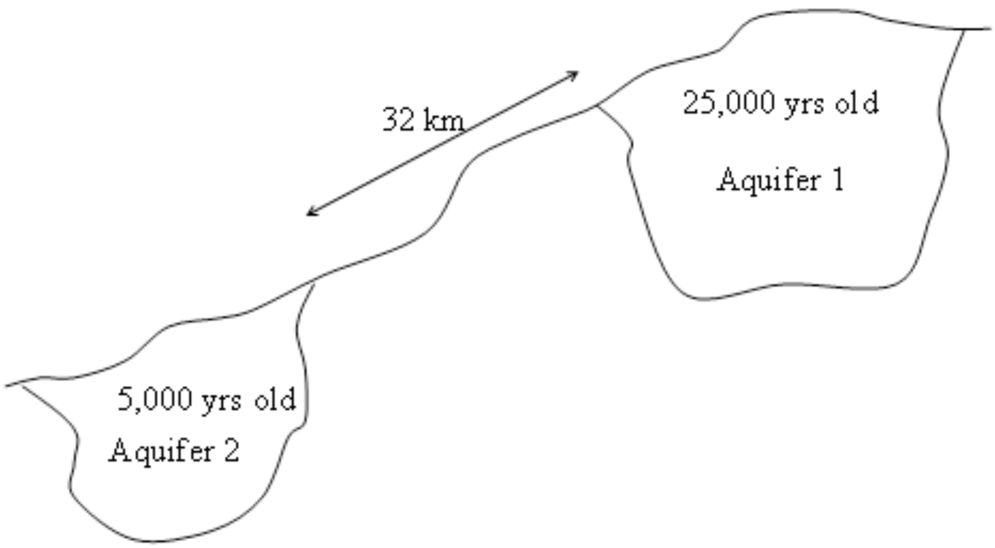


Figure 4: Flow between aquifers

Problem 7 Designing a water filtration system

(Hillel, 1998)

More than a century ago, the inhabitants of the French mustard-making town of Dijon numbered perhaps 10,000. Since they drank mostly wine, their daily water requirement were, say, no more than 20 liters per person. But the quality of the town's water supply was deteriorating badly.

Now supposing we were given the task of purifying that town's water today (with the benefits of hindsight) and we knew that a column thickness of 0.30 m was needed for adequate filtration and that the hydraulic conductivity of the available sand 2×10^{-5} m/sec. Could we calculate the area of filter bed needed under a hydrostatic pressure (ponding) head of 0.7 M ? Consider the flow to be vertically downward to a fixed draining surface.

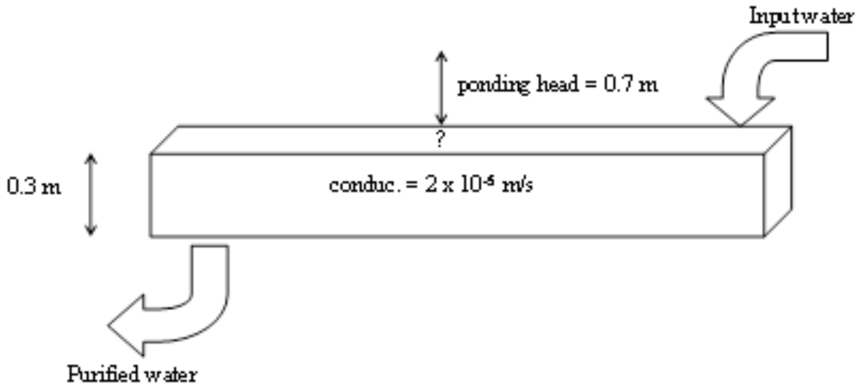


Figure 5: Design of filtration system for Dijon

iv) Heterogeneity

Heterogeneity is not a complexity, but a fact in geology. It arises because of assumptions made in simplifying the basic concepts such as in Darcy's law, conductivity etc. In real field situations, one cannot but ignore the aspect of heterogeneity.

In terms of hydraulic conductivity, some simple formulae are available to quantify the combined effect of layered heterogeneity, in series or in parallel. The concept is similar to that with electrical resistors. But one should keep in mind that even such simple arrangements of geological heterogeneity are seldom found.

'Effective conductivity' is the overall hydraulic conductivity of a medium that is composed of smaller homogenous units with uniform conductivity units. The effective conductivity is not a fixed entity, but changes with the flow condition. For example, the same medium can exhibit different effective conductivity depending on whether the flow is vertical or horizontal.

For a flow occurring through a set of homogenous layers with conductivities K_1, K_2, …, K_n aligned in parallel. The effective conductivity, K_{eff} is given by the arithmetic average,

$$K_{eff} = \frac{1}{n}(K_1 + \ldots + K_n) \qquad \textbf{Equation 6}$$

In case of layers with varied thickness, b_1, …, b_n, the appropriate weighted average should be taken.

For flow through layers in series, the effective conductivity K_{eff} is given by the harmonic average,

$$\frac{1}{K_{eff}} = \frac{1}{n}\left(\frac{1}{K_1} + \ldots + \frac{1}{K_n}\right)$$ **Equation 7**

A similar weighted average should be taken in case of layers with varied thicknesses. In general, for any flow condition, the effective conductivity lies between these two extremes of the arithmetic and harmonic averages.

Problem 8

(Karanth, 1987)

A natural filtration scheme is planned to be designed for a city. As part of this scheme, the wastewater of the city would be let through a lake and then through a confined layered bed of sandy and gravel material. The flow would be almost horizontal through these layers.

The confined bed comprises materials of different hydraulic conductivities, as follows: 5 m of k = 12 m/day, 10 m of 25 m/day and 10 m of 40 m/day. Determine the rate of ground water flow through a kilometre width of this bed under a hydraulic gradient of 1 in 300 of the piezometric surface.

If the waste output from each household is 200 litres/day on average, how many households can this kilometer wide system accommodate? If this system, needs to store water of one year's input, how long would the bed need to be? (Hint: calculate the speed and with that the time of flow in 1 day.)

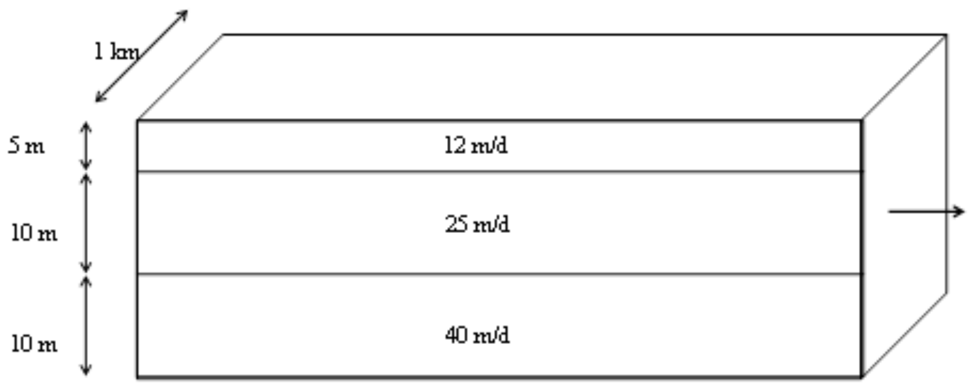


Figure 6: Flow through a multiple layered bed

v) Aquifer Interaction

A unconfined, phreatic or water table aquifer is one that is exposed to the atmosphere and an confined aquifer is one that is under pressure. Often, the distinction between confined and unconfined conditions are not very clear. There is interaction between aquifers due to conducting beds or breaks in separating layers. This results in flow between the aquifers, the direction of flow depending upon the hydraulic gradient across these aquifers. For example, an artesian confined aquifer could leak vertically into a phreatic aquifer. In the same situation, the artesian conditions might change with use of the confined aquifer and there could be reversal of flow from the unconfined down into the confined aquifer.

Problem 9 Leaky aquifer

(Karanth, 1987)

Often, we need to estimate the amount of vertical flow through aquifers separated by semi-permeable layers. One way to do this is to have measurements of hydraulic head in the two aquifers. This is possible if there are observation wells located nearby and tapping the two aquifers. This along with other information such as hydraulic conductivity of the aquifer material can give us an estimate of the flow due to interaction between the aquifers.

The Piezometric surface of a confined aquifer is 2 m below the water table in the unconfined aquifer above. The two aquifers are separated by an aquitard of 3 m thickness. The water lies 15 m above the top of the aquitard k of the aquitard is 0.08 m/day and that of the unconfined aquifer is 1.2 m/day. Determine the recharge rate from the unconfined to the confined aquifer.

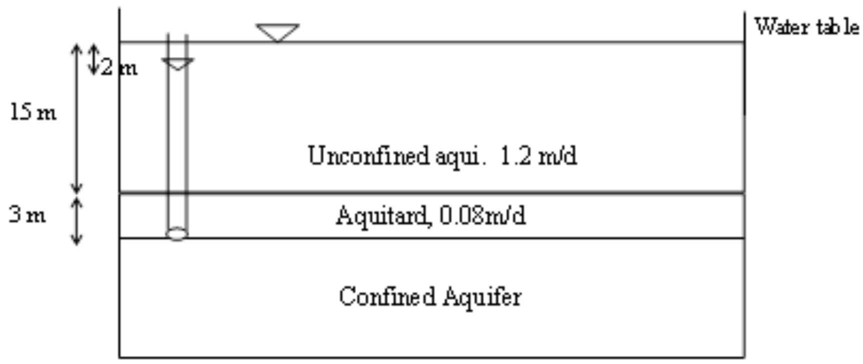


Figure 7: Leakage from unconfined into confined aquifer

Problem 10 Discharging aquifer

(Karanth, 1987)

In case of confined aquifers under high pressure, there can be vertically upwards flow into the unconfined aquifer. Such direction of flow can also change seasonally with recharge/discharge conditions.

In a ground water discharge area, a leaky artesian aquifer is overlain by a water table aquifer. Given that the piezometric head and the water table are, respectively, 20 and 12 m above the top of the artesian aquifer and the vertical hydraulic conductivity of the water table aquifer 1.2 m/day , find the rate of leakage. Also, find the head in a piezometer with its bottom 3 m above the top of the aquifer

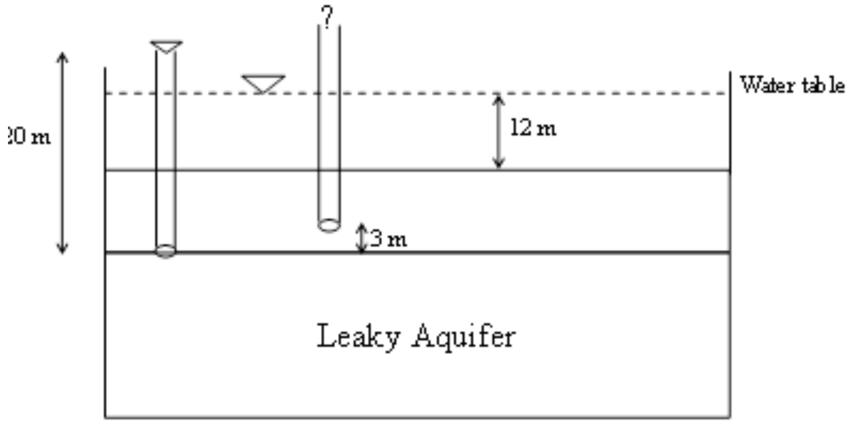


Figure 8: Discharging flow from artesian aquifer into unconfined aquifer

vi) Specific yield, recharge and discharge

Not all the water present in rocks however, flows under gravity. Depending on the degree of connectedness of water-bearing pores and amount of forces such as capillary forces binding water molecules together, only a fraction of the water is available for flow under gravity. For unconfined aquifers, this proportion of water that flows under gravity is called Specific Yield (S_Y). It is defined as the ratio of volume of water drained under gravity to that which is present in the aquifer within the drained volume for a unit drop in water table. The values of S_Y can be around 20% for sand and gravel aquifers and around 1%-5% for clay-dominant formations. Maximum value of S_Y is the porosity.

The specific yield S_Y is the difference between the effective porosity and specific retention of aquifer material.

For confined aquifers, the pressure from overbearing strata can be high. As a result, withdrawal of water from such aquifer results in reduction of pore pressure and compression of the soil/rock matrix and vice-versa. The ratio of amount of water released in an unit volume from unit drop in head in a confined aquifer is the Storage coefficient or Storativity. This change is storage comes about from a compression of the rock matrix during release of water.

The amount of water released or recharged from/into an aquifer is given by:

$$\Delta W = \Delta h . Sy . A \qquad \textbf{Equation 8}$$

where ΔW is the amount of water released/recharged for a drop/rise in water level of Δh over an aquifer with specific yield Sy and area of extent A.

Problem 11

(Karanth, 1987)

The Indus river flows through the alluvial-gravel region of the Himalayas and at different point loses and gains water from streams. In order to look at the base flow component from the aquifer to streams, a small watershed was taken for measurements. In the non-monsoon dry season, the only flow from the aquifer was into the streams. In order to measure the base flow from the aquifer, the fall in groundwater table during this period was recorded.

This unconfined aquifer was of one sq. km extent and had a water-table decline of 2 metres, given the following water bearing and yielding characteristics:

0.8 m-porosity 25%, specific retention 5%
1.2 m- porosity 22%, specific retention 10%

If the total flow in the river during this period was 0.5 MCM arising partly due to base flow and rest from snow-melt, how much proportion of flow in this stream arrived from base flow?

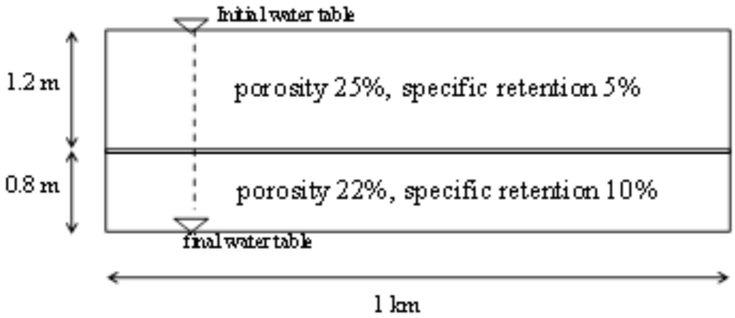


Figure 9: Aquifer draining through material of different storage properties

Problem 12 Recharge estimation

(Karanth, 1987)

A severe drought struck in Northern part of Gujarat, India, resulting in an average decline of 2 m in the water table over an area of 50 sq. km.due to withdrawal of 15×10^6 m^3 of water from the phreatic aquifer during a period of drought. Subsequently, rainfall of 1200 mm occurred and the water levels rose by an average of 1.6 m. Determine the specific yield of the materials in the zone of water-level fluctuation and rainfall-infiltration factor. Assume that the specific yield of the materials is uniform.

A groundwater recharge project planned by a local NGO aims to provide increased groundwater recharge to raise the water table by average of 0.5 m every year. Assuming that 30% of runoff can be recharged, for an average rainfall of 800 mm, how large a catchment area would be needed to provide for this additional recharge?

Problem 13 Post-monsoon rise in water table and recharge

(Karanth, 1987)

For a recharge project, we need to estimate the catchment area needed for achieving a desirable amount of recharge.

The first step is computing the rainfall infiltration factor:

Annual rainfall records available for a station for 10 years. The average pre and post monsoon depth to phreatic aquifer groundwater levels have been recorded only for 5 years.
Compute

(1) minimum rainfall required to effect water level rise (Hint: Plot the rainfall along with water level changes),
(2) average water-level fluctuation for the period of rainfall record,
(3) average recharge, assuming specific yield as 0.15, and
(4) average rainfall-infiltration factor.

Assume population density is 350 persons/km^2. The domestic water requirement is 120 lpcd. Irrigation water requirement is 10 times the domestic water requirement. Then, for each km^2 of this aquifer, how much catchment area is needed to generate sufficient recharge to feed this population? Assume a future average future rainfall of 800 mm.

What is the ratio of catchment area to potentially sustainable populated area?

Year	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
Rainfall (m)	0.97	0.25	0.31	0.58	0.70	0.91	1.13	0.86	0.50	1.19
Water level rise (m)	NA	NA	NA	NA	NA	2.7	3.0	1.45	0.75	3.5

NA = Not Available

Table 4: Rainfall and groundwater level data for 10 years

Problem 14 Return flow from irrigation

(Raghunath, 2002)

In a phreatic aquifer extending over 1 km^2 the water table was initially at 25 m below ground level. Sometime after irrigation with a depth of 20 cm of water, the water table rose to a depth of 24 m b.g.1. Later 3×10^5 m^3 of water was pumped out and the water table dropped to 26.2 m b.g.1. Determine (i) specific yield of the aquifer, ii) Return flow from irrigation (iii) deficit in soil moisture (below field capacity) before irrigation.

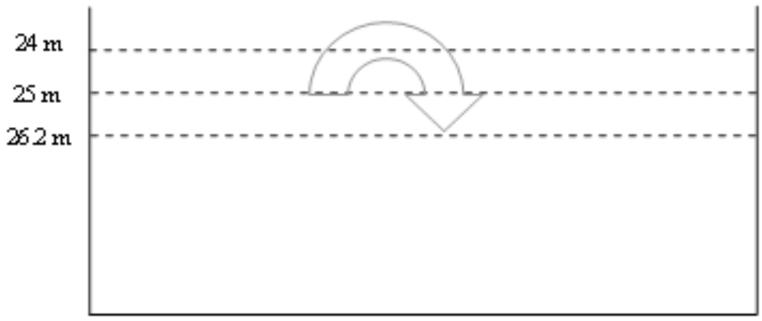


Figure 10: Rise and fall of water table due to irrigation and pumping

Problem 15 Optimal Pumping

(Raghunath, 2002)

In a certain place in Andhra Pradesh, India, the average thickness of the confined aquifer is 30 m and extends over an area of 800 km^2. The piezometric surface fluctuates annually from 19 m to 9 m above the top of the aquifer. Assuming a storage coefficient of 0.0008, what ground water storage can be expected annually ?

Assuming an average well yield of 30 m^3/hr and about 200 days of pumping in a year, how many wells can be drilled in the area ?

Problem 16 Recharge Zone and pumpage

(Raghunath, 2002)

During ground water investigation in the Cauvery basin in India by UNDP the following data water collected.

Recharge area identified	19×13 km
Annual rainfall	1070 mm
Infiltration	20 % of rainfall (approx.)
Transmissibility of the aquifer (from pump tests in the discharge area)	6×10^6 1 pd/m
Width of the aquifer	21 km
Hydraulic gradient (towards the Discharge area from observation wells)	1.14 m/km

It has to be ascertained whether all the pumpage comes from the recharge area.

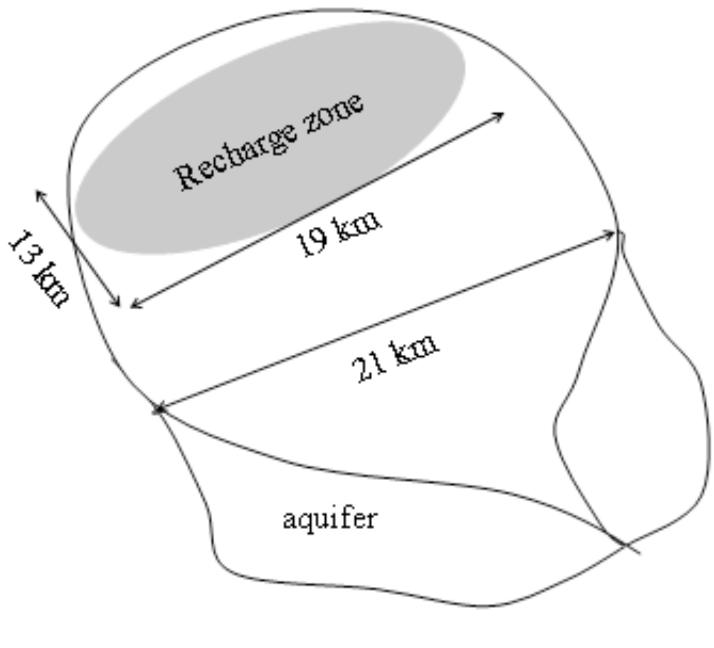


Figure 11: Recharge zone of aquifer

Problem 17

(Raghunath, 2002)

An aquifer has an average thickness of 60 m and an aerial extent of 100 ha. Estimate the available ground water storage if

(a) the aquifer is unconfined and the fluctuation in GWT is observed as 15 m,

(b) the aquifer is confined, and the piezometric head is lowered by 50 , which drains half the thickness of the aquifer.

Assume a storage coefficient of 2×10^{-4} and a specific yield of 16 %.

Problem 18

(Raghunath, 2002)

An artesian aquifer 20 m thick has a porosity of 20 % and bulk modulus of compression 10^8 N/m^2 . Estimate the storage coefficient of the aquifer. What fraction of this is attributable to the expansibility of water.

vii) Well drawdowns

The subject of well hydraulics deals with the changes in groundwater flow caused by pumping. For idealized assumptions, some computable functions are available to estimate the drawdown due to pumping from a well. Most of these functions assume an infinite aquifer horizontally and uniform aquifer properties. Also, the impact of multiple wells can be linearly superposed.

The basic work on unconfined aquifer was initiated by Dupuit in 1848 who gave the equation for drawdown due to pumping from a well. This was later modified by Theim in 1906 to give:

$$K = \frac{2.3Q \log_{10}(r_2/r_1)}{\pi (h_2^2 - h_1^2)}$$

Equation 9

where two points on the cone of depression at radial distances r_2 and r_1 have heads h_2 and h_1 respectively and that steady state condition of water tables is reached under constant pumping at rate Q in a medium of hydraulic conductivity K.

For similar assumptions, a confined aquifer would give,

$$K = \frac{Q \log_{10}(r_2/r_1)}{2.72 Kb (h_2 - h_1)}$$

where b is thickness of the confined aquifer. This equations assume complete penetration of the well within the aquifer.

Problem 19

(Karanth, 1987)

A well in the centre of an unconfined island aquifer, bounded externally by a circle of radius

1200 m, is proposed to be pumped at a rate that will limit the drawdown to 6 m at a distance of 15 m from the well. What should be the maximum allowable discharge of the well, given that the height of the static water table above the impermeable base of the aquifer is 10 m and the hydraulic conductivity of the aquifer is 0.1 m/day.

Problem 20

(Karanth, 1987)

A fully penetrating well of diameter 0.30 m abstracts water from a confined aquifer. Compute the discharge when the steady state drawdown at distances of 10 and 60 m are respectively 2.4 m and 0.5 m, given the thickness of the aquifer and hydraulic conductivity as 20 m and 10 m/day respectively.

viii) Interpreting head data

Much information can be derived by interpretation of well head levels if they are understood properly. The patterns in spatial and temporal head levels are a product of various factors, i) geological heterogeneity, ii) aquifer thickness, iii) leaking into other aquifers, iii) pumping, recharge and discharge.

From Darcy's law, one can infer that if velocity of flow is constant, then the only factor affecting hydraulic gradient and therefore spacing of head contour should be changes in conductivity. This is so in case there is no discharge or recharge within the system and only flow from one point to another.

Often, precautions need to be taken in case of interpretation of piezometric levels towards measuring hydraulic head. The piezometric level coincides with the water table for an unconfined aquifer only when the flow is almost horizontal. Sometimes, local effects such as surface water bodies could produce local flow systems which can be misinterpreted. Therefore, one needs to look for different causing factors apart from the major ones such as topography and geology. In highly forested environments, there could be diurnal variations in the water table caused because of daytime ET. Such effects need to be taken into account. In human modified environments, pumping can be a significant factor and the flow system in many cases can never be assumed to be in a steady state. One important factor when correlating piezometric levels obtained from wells in about ensuring that they belong to the same flow system eq. the same aquifer. Often, there is mixing of data from different aquifers leading to confusion in flow behaviour.

Problem 21 Impact of heterogeneity on head contours

(Domenico and Schwartz, 1990)

If the hydraulic conductivity in area A is 10^{-6} m/sec, determine the hydraulic conductivity in the other areas. Assume the medium is isotropic and nonhomogeneous and that no flow is added to or lost from the system; that is, inflow equals outflow.

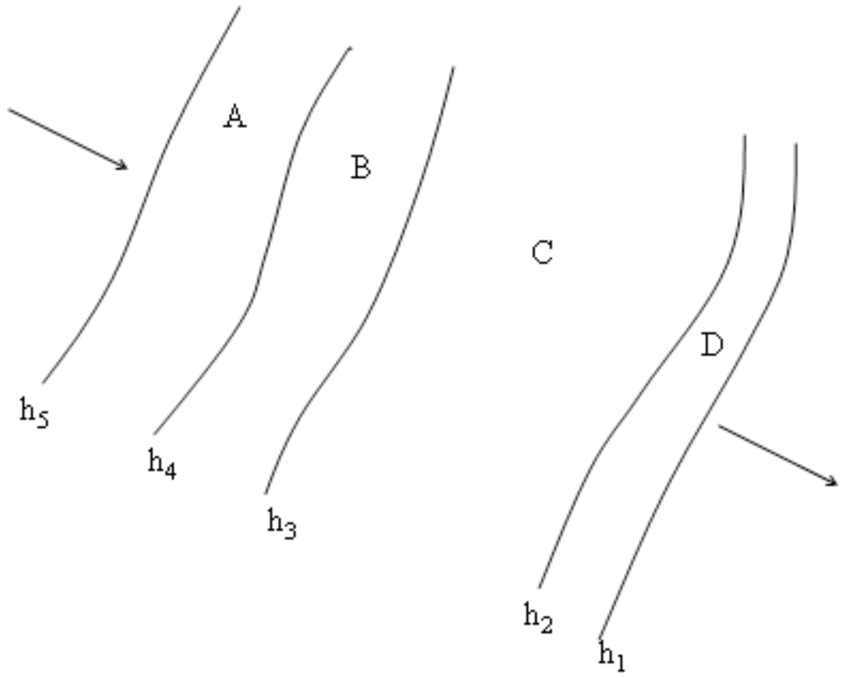


Figure 12: Flow system in heterogenous aquifer

Problem 22 Interpreting flow system from head contours

(Domenico and Schwartz, 1990)

You are given the following piezometric surface:
For each of the conditions cited, give one reason that can account for the piezometric surface

Condition 1: Inflow across h = 80m = outflow across h = 30m
Condition 2: The aquifer is homogenous and isotropic
Condition 3: The aquifer is homogenous and isotropic

The aquifer depicted in the Figure is underlain by a uniformly thick homogeneous clay layer. Below this clay layer is another aquifer. The head in this lower aquifer near the 70-m equipotential line is 60m, or 10m lower than the measured 70-m equipotential line. In the vicinity of the 30-m equipotential line, the head in the lower aquifer is on the order of 25 m, or 5 m lower than the measured 30-m equipotential line. In which of these two regions is the velocity of vertical movement across the clay layer the greatest and why? Which way is the flow directed, upward or downward?

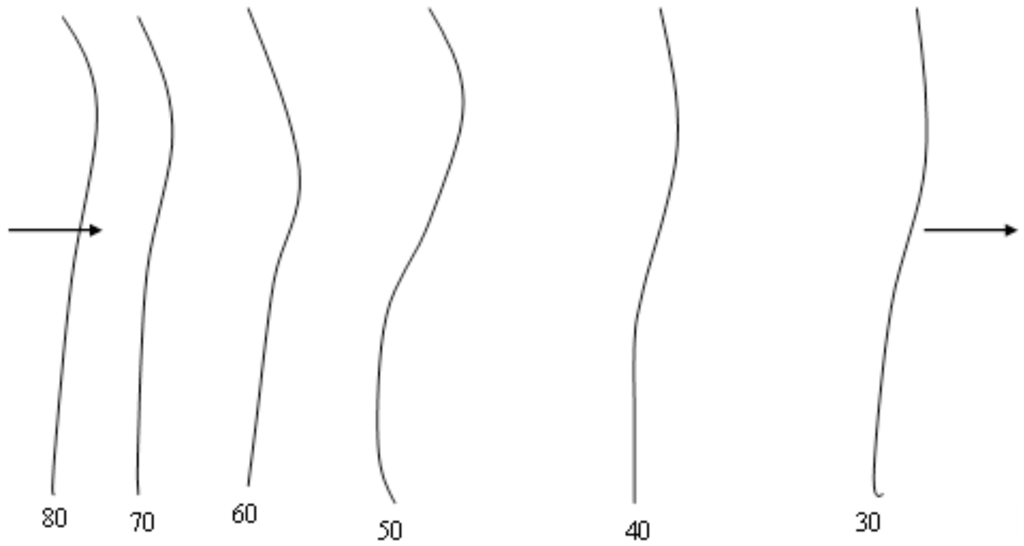


Figure 13: Expanding flow contours

3. Computer Exercises

3.1 Homogenous medium with reclining water table

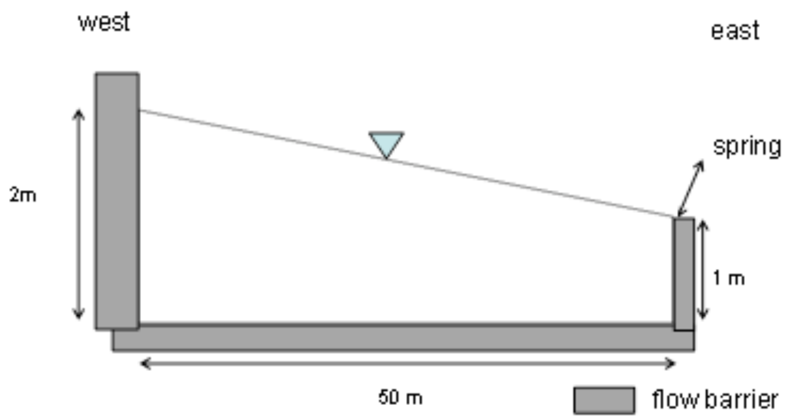


Figure 14: Steady state flow across unconfined aquifer with homogenous properties

An gently sloping watershed slopes from West to East direction. These two barriers can be considered as no-flow boundaries and the only discharge point for this flow system is the spring located in the Eastern boundary from which all discharge occurs. For simplicity consider the deposits between the barriers to be homogenous with uniform and isotropic properties of hydraulic conductivity 2e-5 m/s and porosity 20%. The cross-section profile of the water table in the unconfined aquifer is shown in the Figure. Assume that the flow is bounded at the bottom of this aquifer by an impermeable granite layer with negligible flow through the granite. Use program TopoDrive to check your results. Use 50 x 50 discretization. With model domain length of 50 m and vertical exaggeration of 10.0. Draw water table from 2m to 1m.

i) *Draw the steady state groundwater flow profile through this intermontane valley deposit.*
ii) *What is the maximum travel time taken for any recharge from the high mountains on the West to reach the spring as discharge?* (Hint the animation tool in the program TopoDrive to observe movement of particles starting from West towards East). *If we were to observe the spring discharges, can we assume an annual cycle of recharge and discharge in this intermontane deposit?*
iii) *Calculate the discharge across the central cross-section. Assume for this purpose, uniform gradient across the cross-section. Assume that this discharge occurs finally along the stream. What is the recharge rate across 1m thickness of the aquifer and 1m length of this cross-section. If the recharge coefficient is 0.25, what is the average annual rainfall of this location. If a 1 kilometre cross-section of this aquifer finally discharges through this stream, how many families can this spring system conveniently support?* (Assume 120 litres daily requirement for individual).

Cross-check these flow patterns with those produced by TopoDrive program.

3.2 Heterogenous medium with reclining water table

Now insert a flow barrier of low conductivity 1e-9 m/s as below:

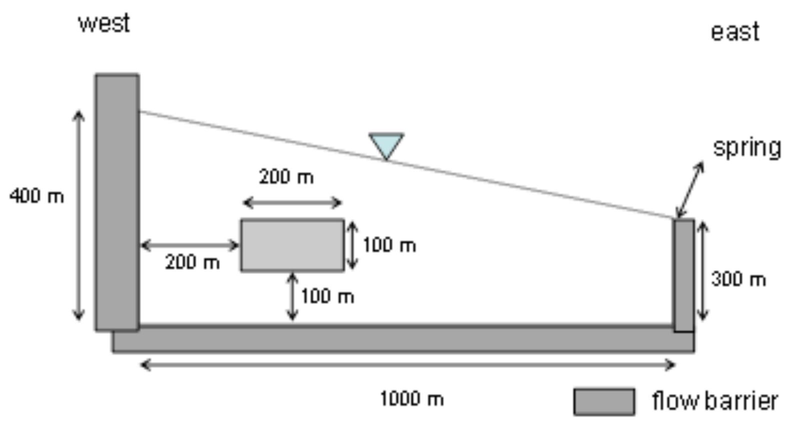


Figure 2: Steady state flow across unconfined aquifer with heterogenous properties

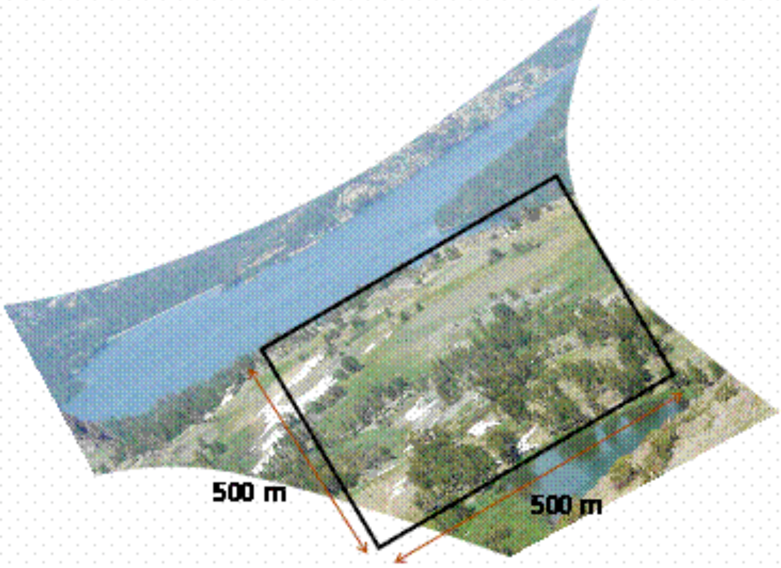
i) *Draw the flow paths for this system. Which are the regions with major changes from before?*
ii) *What is the travel time for the flow paths traversing through the base of the deposits?*

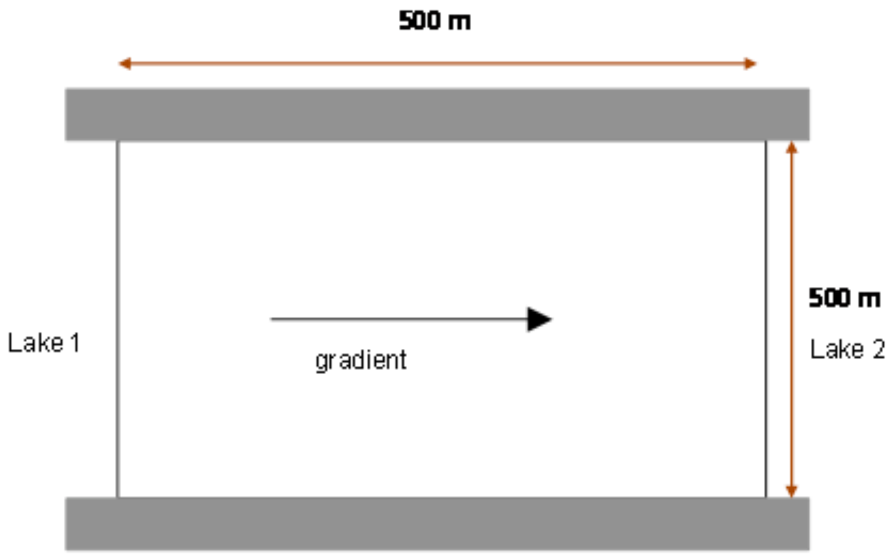
Can we now consider this system as an annual cycle of recharge and discharge? How much proportion of the discharge from the spring has actually recharged more than a year back? In this case, how would the recharge rate in the Western mountains be affected? What does this imply for the number of families this spring system can sustain?

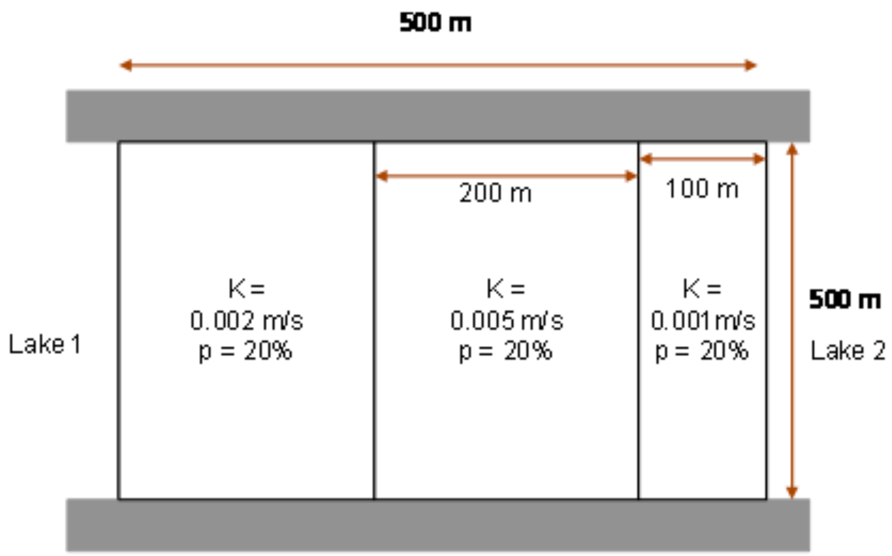
3.3 Two dimensional flow with constant head boundary conditions

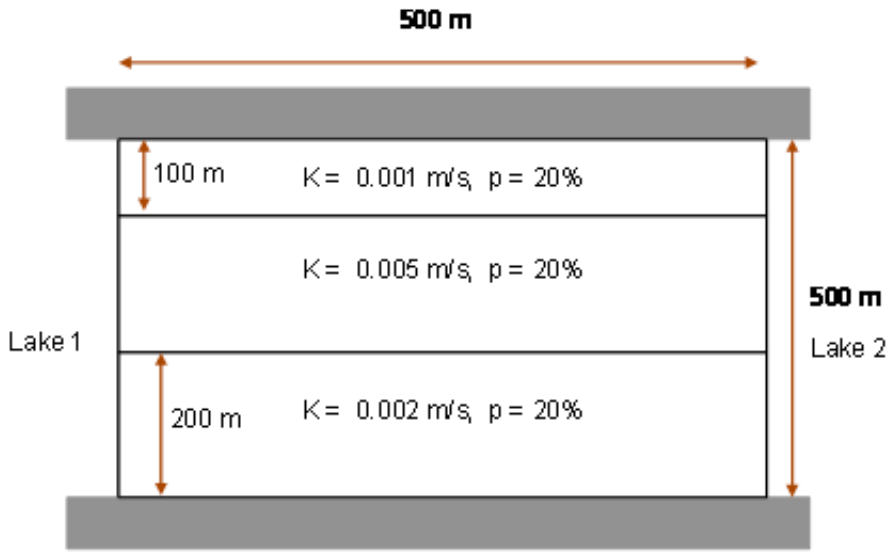
Flow occurs between two lakes through an aquifer connecting them that is bounded for flow on all other sides. We consider two-dimensional flow between the two lakes through this aquifer. The level difference between the two lakes is 50 m.

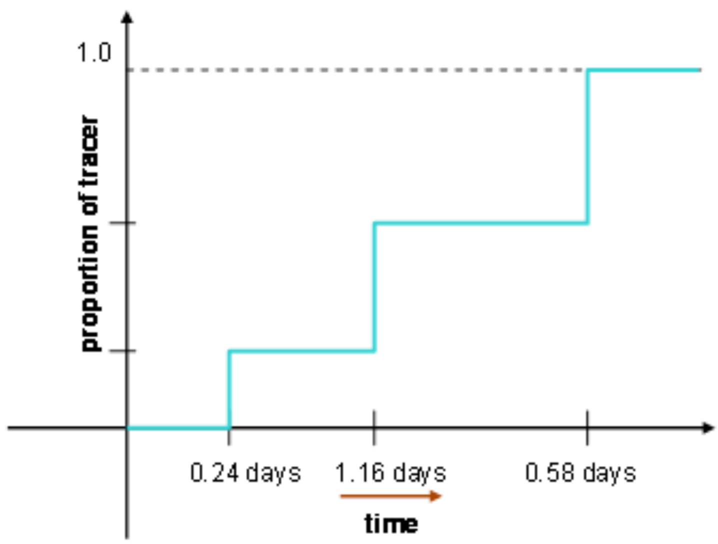
Let the aquifer material be homogenous with conductivity of 0.001m/s and porosity 20%. Calculate the travel time of flow from one lake to another. Check this travel time using the program ParticleFlow.
Now, let the aquifer have layered bands of different conductivity as shown in the Figure. Compute the effective conductivity of this combination. Use program Particle flow and also expression for effective conductivity for flow in series. Now, calculate how much time it takes to travel from one lake to another.
Instead, let the bands be along the flow direction, see Figure. Now, compute the effective conductivity again. How would you use the travel times obtained from Particle flow to compute this same effective conductivity for this combination.
A non-diffusive tracer released uniformly from the first lake is observed at the second lake to reach in spikes as shown in the figure. If porosity is uniform at 20%, guess one possible configuration of hydraulic conductivity pattern within the aquifer using this tracer result. Can there be other possibilities?




500 m
500 m
Lake 1
gradient
Lake 2


500 m
200 m
100 m
500 m
Lake 1
K = 0.002 m/s p = 20%
K = 0.005 m/s p = 20%
K = 0.001 m/s p = 20%
Lake 2





4. Solutions to Numerical problems

Problem 1

a)

Time Year	Recharge from Direct Precipitation m^3	Net Recharge from Stream flow m^3	Discharge by Pumping m^3	Discharge by Evapotranspiration m^3	Change in Ground Water Storage m^3
1	3×10^7	0	0	3×10^7	0
2	3×10^7	6×10^5	1×10^7	3×10^7	94×10^5
:	:	:	:	:	:
7	3×10^7	1×10^6	3×10^7	9×10^6	8×10^6
8	2.8×10^7	2×10^6	3.5×10^7	5×10^6	10×10^6
9	2.5×10^7	3×10^6	3.5×10^7	3×10^6	10×10^6
10	3.5×10^7	4×10^6	4×10^7	1×10^6	2×10^6
11	3.5×10^7	4×10^6	4.2×10^7	1×10^6	4×10^6
12	3.5×10^7	4×10^6	4×10^7	1×10^6	2×10^6

The abstraction from this aquifer exceeds the long term average recharge by 4.5 x 10^6 m^3. Therefore one can say that the current abstraction is reducing the storage of the aquifer. This might result in different adverse consequences such as increase in energy for pumping, greater investment for wells, drying up of wells, increased mineralization of groundwater etc. All these together would determine the degree of *overexploitation* of the aquifer.

b) The proportion of natural recharge available for natural ET changes over the years as 100%, 98%, 29%, 16.7%, 10.7%, 2.6%, 2.6% and 2.6%. All the recharge is now being diverted for pumping.

c) The decline in water levels over the years are 0 m, 9.4 m, 8 m, 10 m, 10 m , 2 m , 4 m , 2 m, i.e an average of 5.6 m every year. This is a rapid decline of water level conditions.

d) In year 8, the net overabstraction is 2 x 10^6 m^3. The pumping would need to be limited to 40-2 = 38 x 10^6 m^3 so that the water level is steady. This however does not change the water available for natural ET which will still be low.

Problem 2

Table 5

Day	**Rainfall**	**Soil Moisture**	**Actual ET**	**Moisture surplus**
1	mm	mm	mm	mm
2	0	100.1	4.3	0
3	45	95.8	6	0
4	26	134.8	6.4	4.4
5	19	150	6.4	12.6
6	19	150	6.4	12.6
7	26	150	6.4	19.6
8	0	150	6.4	0

a)

Rainfall = Actual evapotranspiration + Moisture surplus ± Change in soil Moisture storage (Remaining soil moisture – Antecedent soil moisture)

$$135.0 = 42.3 + 49.2 + 43.5$$

b) The total moisture surplus is 43.5 mm. If 20% of this infiltrates, then the recharge to groundwater is 0.2*43.5 = 8.7 mm. The rainfall-recharge coefficient is therefore 8.7 mm out of 135 mm storm i.e. 6.4%.

Problem 3

Applying the expression $H = H_P + H_E$ we get the following:

Depth of piezo (ft): 600, 575, 150, 25
Total head (ft): 900, <u>800, 700, 600</u>
Pressure head (ft): <u>600, 575, 150, 25</u>
Elevation Head(ft): <u>300, 225, 550, 575</u>

Problem 4

Head1 = Elevation head + Pressure head = 50 cm + 32 cm = 82 cm
Head2 = Elevation head + Pressure head = 30 cm + 26 cm = 56 cm

Q = kiA = 10m/day * (82-56)/50 * p*$(0.1)^2$ = 0.163 m^3/day

Problem 5

Soln: From Darcy's law,

$$v = \frac{Ki}{\theta}$$

Hydraulic gradient

$$i = \frac{dh}{dl} = \frac{164 - 152}{1800} = 12/1800 = 6.67\times 10^{-4}$$

v = 0.073 m/day

Problem 6

It has taken 20,000 years for the ground water movement through the inclined water bearing stratum, to form a recent potential ground water storage.

$$v = \frac{32*1000}{20000*365} = \text{m/d}$$

Applying Darcy's law,

$$0.00438 = K\frac{0.2}{1000}$$

K = 21.90 m/day

Transmissibility of the water bearing stratum,

$T = K.\ b = 21.90 \times 30$
$= 657\ m^2/day$ or $m^3/day/m$
$= 657{,}000$ 1pd/m, or $657\ m^2/day$
.

Problem 7

Soln: The flow required is $10{,}000 \times 20 \times 10^{-3}\ m^3$ / day i.e. $200\ m^3$ / day
The velocity through this sand is $v = K . I = 2 \times 10^{-5}$ m/sec x 0.7/0.3 m/sec = 4.032 m/day
$A = Q/v = 200 / 4.032 = 49.6\ m^2$ of area required i.e. a 7 m x 7 m area pit with depth of 0.3 m

Problem 8

Soln: Effective conductivity for flow in horizontal direction is:
(5 x 12 + 10 x 25 + 10 x 40)/(5 + 10 + 10) = 28.4 m/d

Q = KIA = 28.4m/d * 1/300 * 1000 m = 94.67 m^3/d

At the rate of 200 litres per household, we can accomodate 473 households.

The distance traveled by this flow in 1 day is 28.4 * 1/300 = 9.5 cm. In 365 days, it would travel, 34.55 metres. Assuming that the output water is discharged after 1 year, one can flush out the waste water through this system. However, note that the input water would create a new head gradient and alter the previous flow. So in actual, the bed system would need to be longer. This can be determined by solving for a constant flow boundary in a confined aquifer system.

Problem 9

18/K = 15/1.2 + 3/0.08
K = 0.36 m/day
i = 2/18 = 1/9
v = Ki = 4 cm/day

Problem 10

Soln:
$q_{leak} = K (h_1 - h_2)/h_2 = 1.2 * (20 - 12)/12 = 0.8$ m/day
The vertical hydraulic gradient is (20-12)/12 = 0.667 m/m. For a point 3 m above top of the arterian aquifer, the head drop will be 3*2/3 = 2m, i.e. at 2m below ground level.

Problem 11

Soln: Water drained = Area * Water level fall * Specific Yield. The water drains through two layers with different specific yields of (25% - 5% = 20%) and (22% - 10% = 12%) respectively. The total water drained = 1sq. km. * 0.8 m * 0.2 + 1sq. km. * 1.2 m * 0.12 = 0.16 sq. km. m + 0.144 sq.km. m = 0.304 sq. km. m. = 0.304 MCM. The proportion of base flow is 60%.

Problem 12

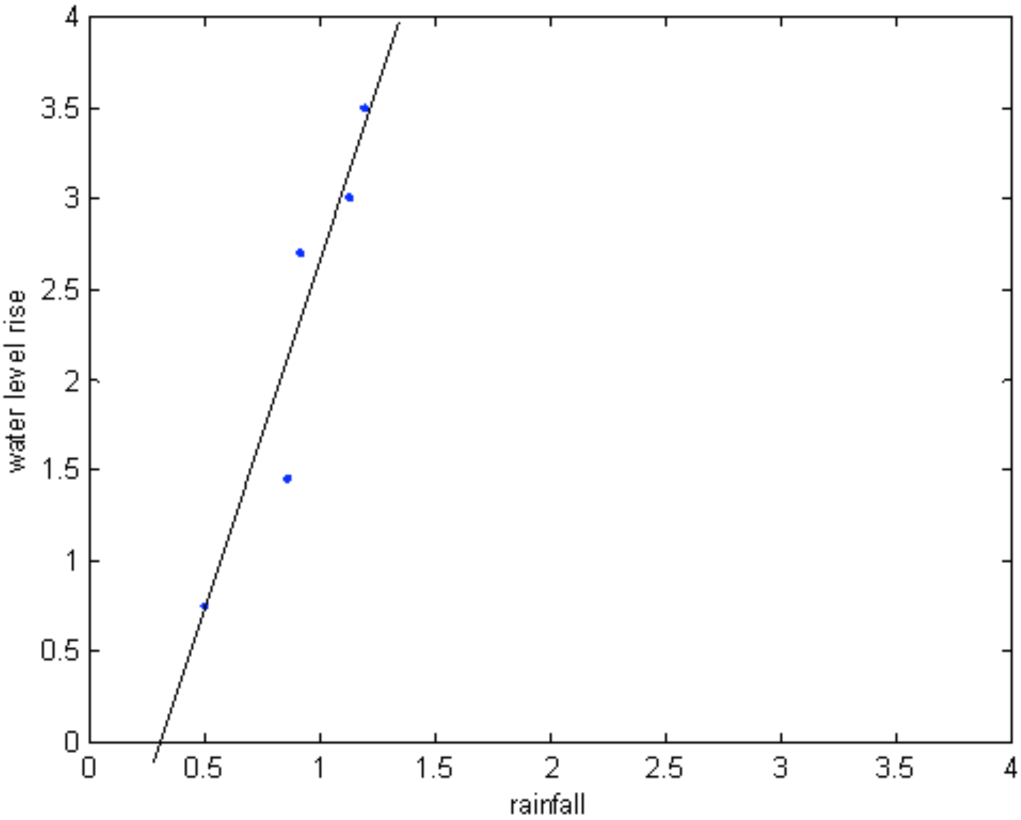
$Q = A. Sy . H$
$1.5 * 10^6 = 50 * 10^6 . Sy . 2$
$Sy = 0.15$

$Rf = H. Sy /R = 0.2$

An additional recharge of $0.375 * 10^6$ m^3 is aimed, This needs a runoff of $1.25 * 10^6$ m^3. for a rainfall of 800 mm, a catchment area of 156.25 hectares are needed.

Problem 13

a) From the graph, the minimum rainfall required for rise in water level is around 0.3 m, 300 mm. This is true for several arid areas with deep water table.
b) Average water table fluctuation is 0.92 m from 1980 till 1984.
c) Average recharge is 0.34 m from 1980 till 1984.
d) Taking the rainfall-infiltration each year and taking average of that we get, average rainfall infiltration factor is 35%.
.



Total annual domestic water requirement/km^2 = 120 l * 350 *365 = 15330 m^3. The irrigation water requirement is 153300 m^3. The rainfall needed to affect this recharge is (153300 + 15330)*100/35 = 481800 m^3. With an average annual rainfall of 800 mm, this requires a catchment runoff generation area of 6.02 km^2.

This means that the population over each piece of land is utilizing groundwater recharge generated over 6 times as much an area.

Problem 14

Solution:
Volume of water pumped out = Area of aquifer × drop in g.w.t. × specific yield

$$3 \times 10^5 = 10^6 \times 2.2 \times S_y$$

$$S_y = 0.136, \text{ or } 13.6\ \%$$

Volume of irrigation water recharging the aquifer = Area of aquifer × rise in g.w.t. × S_y

Considering an area of 1 m^2 of aquifer,

$$1 \times y = 1 \times 1 \times 0.136$$

Recharge volume (depth) y = 0.136 m, or 136 mm

Soil moisture deficit (below field capacity) before irrigation = 200-136
= 64 mm

Problem 15

Solution:

$$\Delta GWS = A_{aq} \times \Delta\text{piezo. Surface} \times S = (800 \times 10_6)(19-9)\ 0.0008$$

Or $6.4\text{-}10^6\ m^3$, or $6.4\ m^3$

Annual draft = (30 × 24) 200 = $0.144 \times 10^6\ m^3$

Number of wells that can be drilled in the area $= \dfrac{6.4}{0.144}$

= 44.5, say 44 wells

of course, the well sites have to be investigated and these should be sufficient spacing for the wells.

Problem 16

Solution:

$$\text{Annual recharge} = (19 \times 13)\ 10^6 \frac{20}{100} \times 1.07 = 5.29 \times 10^7\ m^3$$

$$Q = Tiw = (6 \times 10^3) \frac{1.14}{1000} \times 21{,}000$$

$$= 144 \times 10^3\ m^3/\text{day}$$

Annual pumpage = $(144 \times 10^3)\ 365 = 5.25 \times 10^7\ m^3$

Thus the entire pumpage comes from the recharge area.

.

Problem 17

Solution:

(a) $\Delta GWS = A_{aq}\ .\ \Delta GWS.\ S_{y\,=}\ 100\text{ ha} \times 15\text{ m}\ (0.16)$

$= 240$ ha-m

(b) $\Delta GWS = A_{aq}[\Delta\text{ piezo. Head} \times S + \Delta\text{ GWT} \times S_Y]$

(as confined) (as unconfined)

$= 100\text{ ha}\ [20\ (2\times10^{-4}) + 30\ (0.16)]$

$= 480$

Problem 18

Solution: $S = \gamma_w b\,(\alpha + n\beta)$

$$= 9810 \times 20 \left(\frac{1}{10^8} + 0.20 \times \frac{1}{2.1 \times 10^9}\right)$$

$$= (1.962 + 0.0187)\, 10^{-3}$$

$$= 1.98 \times 10^{-3}, \qquad \text{say, } 2 \times 10^{-3}$$

The fraction of storage attributable to the expansibility of water (taking only the second term within the brackets)

$$S_w = 0.0187 \times 10^{-3}$$

$$= \frac{1.87 \times 10^{-5}}{1.98 \times 10^{-3}} \text{ of S}$$

$$\approx \frac{1}{100} \text{ of S, or 1 \% of S}$$

Problem 19

Soln: Applying Theim equation for unconfined aquifers:

$Q = \pi K(h^2_2 - h^2_1)/(2.3 \log(r_2/r_1)) = 6.03\ m^3/day$

Problem 20

Applying Theim equation for confined aquifers:

$Q = 2\pi Kb(s_1 - s_2)/(2.3 \log(r_2/r_1)\,) = 1334\ m^3/day$

Problem 21

In region B, the equipotential lines are spaced similarly apart as in region A, therefore, $K_1 i_1 = K_2 i_2$ gives the same K i.e. 10-6 m/s. For region C, the equipotential lines are spaced half as wide as in region A, thereore the gradient is half slower, making K twice i.e. 2×10^{-6} m/s. In region D, the equipotential lines are spaced twice closer as in region A, making the gradient twice faster, the K therefore is 5×10^{-7} m/s.

Problem 22

Condition 1: If the inflow and outflows are the same and the aquifer thickness is also the same, then the cause for hydraulic gradient decreasing towards right should be because of increasing hydraulic conductivity in that direction

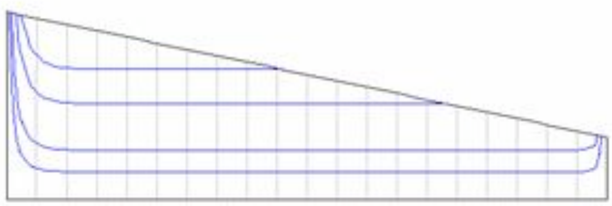
Condition 2: If the aquifer is homogenous and isotropic, then we could have one case where inflow is not equal to outflow. In that case decrease in hydraulic gradient could be due to leakage from the aquifer, maybe to a lower aquifer

Condition 3: In this case, we can consider the aquifer thickness to be increasing from left to right, therefore causing a decrease in rate of flow, therefore, hydraulic gradient.

The vertical hydraulic gradient is greater (10m) at the 70-m equipotential line than at the 30-m equipotential line (5m). Therefore assume other properties such as aquifer properties and clay properties to be similar, we can deduct that the rate of vertical leakage is downward and greater at the 70-m equipotential line.

5. Solutions to Computer Exercises

Solution to 3.1



Taking a 50 x 50 discretization.

ii) Approx in 360 days, most of the discharge has occurred. Yes, for practical purpose, we can consider this system as recharging and discharging annually.

iii) Assume that the flow gradient of (2-1)/50 i.e. 0.02 m/m is present at the central cross-section with vertical area of $1.5m^2$.

q = kiA = 2e-5 * (2-1)/50 * 1.5 m^3 / s for 1 metre of thickness

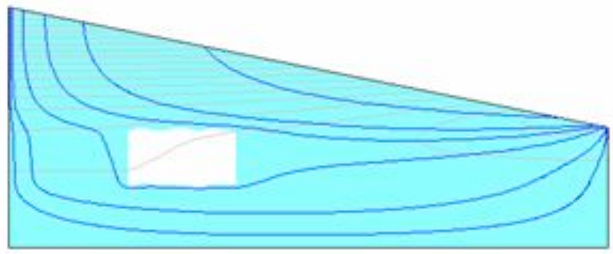
= 6e-7 m^3 / s for 1 metre of thickness. = 6e-4 litres/s.

The total recharge per unit length across this cross-section with 1m thickness contributing to this flow is 6e-7/50m = 12e-6mm/m^2/s. This corresponds to 1.02 mm/m^2/day of recharge per unit area. If the recharge coefficient is 0.25 the average rainfall contributing to this recharge is 1.02/0.25 = 4.08 mm/day which comes to 1489 mm average annual rainfall.

Taking 120 lpcd as the water requirement/person, we have 600 litres of daily requirement for a family of 5. The total discharge per day is 6e-4*60*60*24 = 51.84 litres/metre thickness/day. For a kilometer long thickness, it can support 51.84*1000*/120 = 432 families.

Solution to 3.2

i)



The region around the flow barrier show maximum difference from before. The flow paths now swirl around this barrier.

ii) The spring discharge passing through the lower deposits now taken around 450 days to reach the spring. The system can no longer be considered as recharging-discharging in an annual cycle. This slower flow process through the aquifer also affects the recharge rate. The recharge rate through the Western mountains would now be reduced

Solution to 3.3

a) Flow velocity $v = Ki/p = 0.001*1/0.2 = 0.005$ m/s; Time taken to traverse 500 m is 500 m / 0.005 m/s = 1.16 days. This is cross-check by using flow line tracking within ParticleFlow.

b) For flow in series, effective conductivity is the harmonic average:

$1/K_{eff} = 0.4/0.002 + 0.4/0.005 + 0.2/0.001$

Giving, $K_{eff} = 0.00208333$ m/s

Going back from time of travel calculation using ParticleFlow, we get,

Travel time $t_{eff} = 0.568$ days,

$T = s/v = 500/v$, giving $v = 880.2816$ m/d

$V = K_{eff}\, I/p = K(1)/0.2$, giving, $K_{eff} = 0.002037$ m/s approximately equal to above calculation.

c) The time of travel through each layer is different. A set of particle released uniformly would reach in spikes at the other lake. The travel times observed from Particle flow are
$t1 = 1.158$ days ; $t2 = 0.232$ days ; $t3 = 0.579$ days

Total flow $Q = v. A = v1. A1 + v2.A2 + v3.A3$
$s. A/t = s1 . A1/t1 + s2 . A2 / t2 + s3. A3/t3$
The travel distances are the same,
So, we get, $A/t = A1/t1 + A2 / t2 + A3/t3$
$1/t = 0.2/t1 + 0.4/t2 + 0.4/t3$
$= 0.1727 + 1.7241 + 0.691$
$t = 0.3864$ days

From effective conductivity calculation,

K_{eff} = Arithmetic mean for flow in parallel, giving,

$K_{eff} = 0.2 * 0.001 + 0.4 * 0.005 + 0.4 * 0.002$

= 0.003 m/s

V = KI/p = 0.015 m/s

T = s/V = 0.3858 days, approximately equal to calculation from ParticleFlow

d) The tracer arrives in spikes as in the example in c). Therefore, there must be parallel layers with different conductivities. The travel time for arrival are:
t1 = 0.24 days, t2 = 1.16 days and t3 = 0.58 days, giving velocities of
v1 = 0.02411 m/s, 0.004989 m/s and v3 = 0.009978 m/s respectively.
The conductivies are:
K1 = 0.00488 approx 0.005 m/s
K2 = 0.0009978 approx 0.001 m/s
K3 = 0.0019955 m/s, approx, 0.002m/s

Each of thickness, T1 = 100 m, T2 = 200 m and T3 = 200 m. These can be ordered in any particular fashion, i.e. they could be divided into as many parts and interspersed between each other. It only matters that they are aligned in parallel finally, but their orientiation in North-South cannot be determined from this data alone.

6. References

Karanth, K. R., 1987, Groundwater assessment, development and management, Tata-McGraw Hill Publishing Company Limited, New Delhi

Hillel D., 1998, Environmental Soil Physics, Academic Press

Domenico A. A. and F. W. Schwartz, 1990, Physical and Chemical Hydrogeology, John Wiley and Sons

Hsieh, P.A., 2001, TopoDrive and ParticleFlow—Two Computer Models for Simulation and Visualization of Ground-Water Flow and Transport of Fluid Particles in Two Dimensions: U.S. Geological Survey Open-File Report 01-286, 30 p

Raghunath, H. M., 2002, Groundwater, New Age International Publishers, New Delhi

7. Instructions and links for Softwares

The softwares TopoDrive and ParticleFlow for use in the exercises and their documentation are available at:

http://water.usgs.gov/nrp/gwsoftware/tdpf/tdpf.html