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EFFECT OF ANISOTROPY ON SEEPAGE FROM A WATER BODY

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PREFACE

Assumption of isotropy of the porous medium is commonplace in solving groundwater flow problems. However, it may not be always advisable to rely up on such simplifying assumption as field experience shows that, soils are anisotropic to certain extent. In groundwater flow modelling practices also, anisotropic aquifer systems are modelled as near-isotropic ones. A reason behind this tendency is due to the fact that most of the commercially available groundwater flow models are based on rather simplistic assumptions of isotropy or pseud-anisotropy. Further, parameters like the components in the hydraulic conductivity tensor are to be estimated from the field in case of anisotropic flow computations. Nevertheless, approximating an anisotropic medium to an isotropic one may introduce errors in the computation of flow and estimation of groundwater balance. The present report summarises relevant aspects on the theory of anisotropic flow in porous media and reviews methodology for computing hydraulic potentials in such a medium. Further, computation of hydraulic potentials are carried out in variety of unconfined aquifer systems with different orientations of the soil strata and levels of anisotropy. This report is deemed to be of worth as investigations on hydraulic potentials or flow in anisotropic aquifer systems are not abundant in the literature. It is hoped that groundwater hydrologists may find it useful.

The study has been conducted and reported by Mr. Mathew K. Jose with the guidance of Dr. G.C. Mishra as part of the work programme of the Ground Water Modelling and Conjunctive Use Division of the Institute during 1998-99.

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ABSTRACT

Most of the theoretical analyses of groundwater flow problems are based upon the assumption of isotropy and homogeneity of the porous medium. But field experience reveals the fact that soils are anisotropic to some extent, in general. Anisotropy can be due to stratification in an aquifer which might have taken place as a result of particle orientation during formative stages. Quite often, an anisotropic aquifer system is approximated to an isotropic one and solutions are attempted using flow models which generally assume a Cartesian coordinate system. However, anisotropic flow systems can not be solved with the Cartesian coordinate domain. Approximating an anisotropic medium to an isotropic one may introduce errors in the computation of flow and/ or heads as groundwater flow through anisotropic soils is complex by nature. Besides, general rules applicable to methods for isotropic conditions like flow-net analysis are no longer valid in an anisotropic medium since the directions of flow and hydraulic gradient in such a medium need not be parallel as in the case of an isotropic aquifer. Review of literature shows that investigations on hydraulic potentials or flow in anisotropic aquifer systems are not extensive. This report summarises related aspects on the theory of anisotropic flow in porous media and reviews methodology for computing hydraulic potentials in an unconfined aquifer system. An algorithm has been devised by applying appropriate transformation techniques for anisotropic domain and analytical results. Numerical experiments have been performed using the algorithm to compute hydraulic potentials in certain hypothetical anisotropic aquifer systems. A number of cases have been studied with different coefficients of anisotropy for the aquifer as well as angles inclination of the bedding planes of the soil strata. The simulated hydraulic potentials in the anisotropic domain are depicted as equipotential lines in vertical sections.

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1.0 INTRODUCTION

Generally, theoretical analyses of groundwater flow problems assume the porous medium to be isotropic and homogeneous with respect to hydraulic conductivity. But, field experience and laboratory tests indicate that most soils are anisotropic to some degree. Layered soil formations like sedimentary rocks and loess exhibit anisotropic behaviour. Stratification in such formations may have resulted from particle orientation.

The effect of anisotropy on groundwater flow through certain geologic formations such as alluvial and sedimentary soils has been of particular concern to groundwater hydrologists as the directions of flow and of the hydraulic gradient in an anisotropic porous medium are not parallel (*Marcus, 1962*).

1.1 BASIC CONCEPTS

In groundwater problems the soil body is considered to be a continuous medium of many interconnected openings which serve as the fluid carrier (*Harr, 1962*). Darcy's law governs fluid flow in the porous medium which is represented as:

$$q = -K \frac{\Delta\phi}{\Delta s} \tag{1.1}$$

where q [LT^{-1}] is the specific discharge and $\Delta\phi/\Delta s$ is the hydraulic gradient due to change in hydraulic potential, ϕ over a distance, s. K [LT^{-1}] is called the coefficient of permeability or *hydraulic conductivity*, which is a function of the intrinsic permeability of the medium (k), fluid density (ρ), dynamic viscosity of the fluid (μ), and acceleration due to gravity (g) and is related by:

$$K = \frac{k\rho g}{\mu} \tag{1.2}$$

Hydraulic conductivity values usually show variations through space within a geologic formation. They may also show variations with the direction of measurement at any given point. The first property is termed *heterogeneity* and the second *anisotropy*. Thus, if the hydraulic conductivity is independent of the direction of the velocity, the soil is said to be an *isotropic* flow medium. Moreover, if the soil has the same hydraulic conductivity at all points within the region of flow, the soil is said to be *homogeneous and isotropic*. If the hydraulic conductivity is dependent on the direction of the velocity and if this directional dependence is the same at all points of the flow region, the soil is said to be *homogeneous and anisotropic*. However, when the directional dependence is varying at different points of the flow region, the soil is said to be *heterogeneous and anisotropic*.

Manifestation of inhomogeneity in soil formations may vary in its characteristics and pattern. For instance, a homogeneous soil block may exhibit anisotropic behaviour whereas a heterogeneous soil formation can be isotropic to flow. Therefore, to completely describe the nature of the hydraulic conductivity in a geologic formation, it is necessary to use two adjectives, one dealing with heterogeneity and one with anisotropy. The different possible combinations of heterogeneity and anisotropy can be classified into (*Freeze and Cherry, 1979*): (i) Homogeneous and isotropic, (ii) Homogeneous and anisotropic, (iii) Heterogeneous and isotropic, and (iv) Heterogeneous and anisotropic.

Consider a two dimensional vertical section through an anisotropic soil formation. If α be the angle between the horizontal axis and the direction of measurement of a K value at some point in the domain, then $K = K(\alpha)$. The directions in space corresponding to the angle, α at which K attains its maximum and minimum values are termed as the *principal directions of anisotropy* and they are always perpendicular to one another. In three dimensions, if a plane is taken perpendicular to one of the principal directions, the other two principal directions are the directions of maximum and minimum in that plane.

If an (x,y,z) coordinate system is set up such that the coordinate directions coincide with the principal directions of anisotropy, the hydraulic conductivity values in the principal directions can be specified as K_x, K_y, and K_z. In an isotropic formation $K_x = K_y = K_z$ at any point whereas an anisotropic formation will have $K_x \neq K_y \neq K_z$. If the geology is such that it is not possible to align the principal directions of the hydraulic conductivity values with the rectangular coordinate system, then the hydraulic conductivity tensor is a matrix of nine elements viz., $[K_{ij}]$ for i = x,y,z and j = x,y,z.

2.0 OBJECTIVES

The objective of the present study is to simulate the hydraulic potentials in an unconfined anisotropic aquifer due to a point source of strength, Q [L^3T^{-1}]. The hypothetical aquifer system is formed by a number of layers inclined at angle, α with the horizontal. The principal directions of anisotropy are aligned with the bedding plane of the strata (with hydraulic conductivity, K_1) and perpendicular to it (with hydraulic conductivity, K_2) respectively. Simulation of hydraulic potentials are carried out for varying coefficients of anisotropy ($\sqrt{K_1/K_2}$) in the aquifer and different orientations (α) of the strata. Within the frame of the present investigations, following are the studies being carried out:

(i) Simulation of hydraulic potentials in an anisotropic, single-unconfined aquifer due to a point source.

(ii) Simulation of hydraulic potentials in a layered-heterogeneous aquifer (where each layer is homogeneous and isotropic) due to a point source.

3.0 REVIEW OF THEORY

A survey of the literature reveals that investigations on anisotropic aquifer systems are meagre. A few theoretical results have been reported from earlier studies. The theory of flow of fluids through anisotropic porous medium is presented by Scheiddegger (*1957*), Polubarinova-Kochina (*1962*), Harr (*1962*), and Marcus (*1962*). Some investigations on the transformation of anisotropic medium to isotropic medium are also available (*Bhattacharya and Patra, 1968; Mishra, 1972; and Strack, 1989*).

In numerical groundwater modelling practices, coordinate rotations are effected so that the off diagonal components of the hydraulic conductivity tensor go to zero within grid elements or cells. This is accomplished by defining a global coordinate system for the entire problem domain and local coordinate systems for each cell in the grid (*Anderson and Woessner, 1991*). It is possible to derive equations relating the principal components of hydraulic conductivity defined in the local coordinate system to the components of hydraulic conductivity tensor defined in the global coordinate system (*Bear, 1972*).

Some important characteristics of anisotropic aquifer systems are presented below. This includes the evolution of hydraulic conductivity tensor, basic transformation of the anisotropic domain, relationship between layered heterogeneity and anisotropy, and the nature of anisotropic hydraulic conductivities and potentials in the medium.

3.1 ANISOTROPIC HYDRAULIC CONDUCTIVITY

In a homogeneous aquifer the hydraulic conductivity, K [LT^{-1}] is same in all directions. However, homogeneous aquifers in the true sense are rare and in practice, the soil often is layered with the hydraulic conductivity being different in the directions parallel and normal to the layers. This is illustrared in Figure-3.1. The hydraulic conductivity parallel to the layer, K_1 (in the direction of x^*-axis) being larger in magnitude than that perpendicular, K_2 (in the direction of y^*-axis) to the bedding plane. Let α be the angle of dip (the angle, the bedding plane makes with the the x-axis, in the rectangular Cartesian coordinate system). Let, also, (x, y) and (x^*, y^*) be the Cartesian coordinates of an arbitrary point R in the actual plane and the rotated plane respectively.

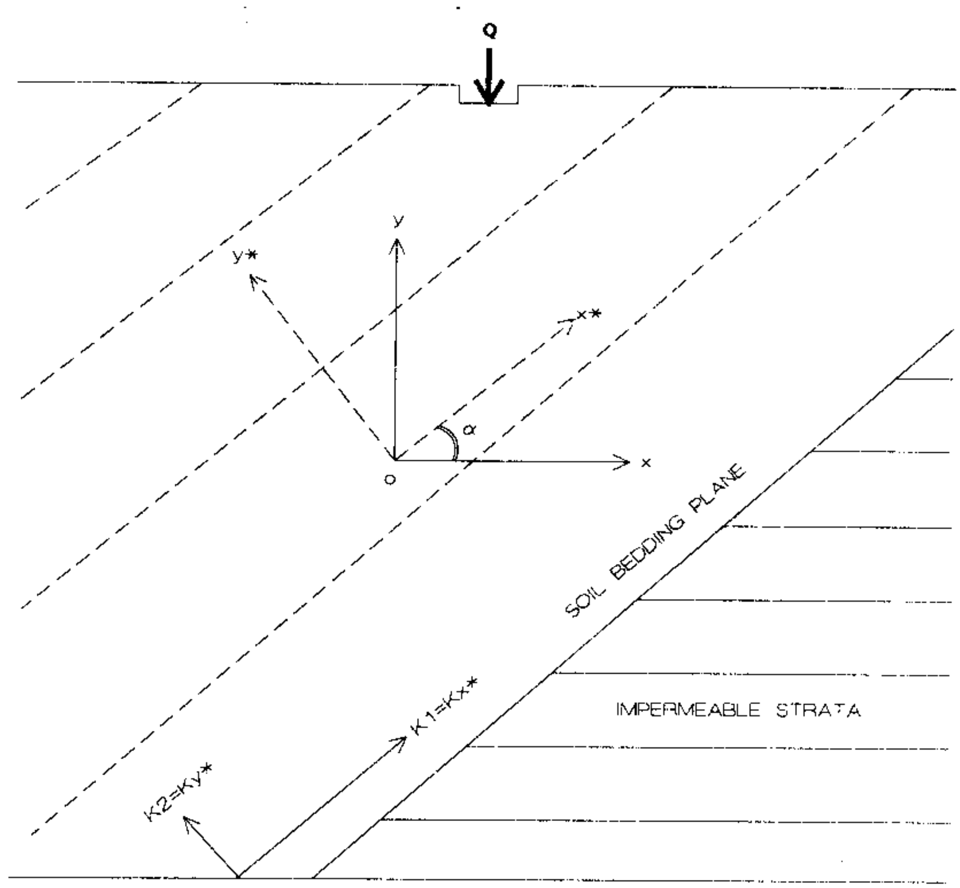


Fig. 3.1 Schematic representation of the hypothetical anisotropic aquifer with inclined bedding planes; the aquifer is recharged from top by a point source Q.

Then,

$$x = x^* \cos\alpha - y^* \sin\alpha$$
$$y = x^* \sin\alpha + y^* \cos\alpha \qquad (3.1)$$

Let q_x, q_y and q^*_x, q^*_y be the corresponding specific discharge vectors in the actual and rotated planes. The expressions for q_x and q_y in terms of q^*_x and q^*_y are similar to eqn.(3.1):

$$q_x = q_x^* \cos\alpha - q_y^* \sin\alpha$$
$$q_y = q_x^* \sin\alpha + q_y^* \cos\alpha \quad (3.2)$$

Now, application of Darcy's law in terms of x*, y* coordinate system yields:

$$q_x^* = -K_1 \frac{\partial\phi}{\partial x^*}$$
$$q_y^* = -K_2 \frac{\partial\phi}{\partial y^*} \quad (3.3)$$

where K_1 and K_2 are the principal values of the hydraulic conductivity.

Using eqn.(3.3) in eqn.(3.2), we get:

$$q_x = -K_1 \frac{\partial\phi}{\partial x^*} \cos\alpha + K_2 \frac{\partial\phi}{\partial y^*} \sin\alpha$$
$$q_y = -K_2 \frac{\partial\phi}{\partial x^*} \sin\alpha - K_2 \frac{\partial\phi}{\partial y^*} \cos\alpha \quad (3.4)$$

By the application of chain rule to eqn.(3.1) yields:

$$\frac{\partial\phi}{\partial x^*} = \frac{\partial\phi}{\partial x}\frac{\partial x}{\partial x^*} + \frac{\partial\phi}{\partial y}\frac{\partial y}{\partial x^*} = \frac{\partial\phi}{\partial x}\cos\alpha + \frac{\partial\phi}{\partial y}\sin\alpha$$
$$\frac{\partial\phi}{\partial y^*} = \frac{\partial\phi}{\partial x}\frac{\partial x}{\partial y^*} + \frac{\partial\phi}{\partial y}\frac{\partial y}{\partial y^*} = -\frac{\partial\phi}{\partial x}\sin\alpha + \frac{\partial\phi}{\partial y}\cos\alpha \quad (3.5)$$

Combining eqn.(3.4) and eqn.(3.5), the Darcy's law for anisotropic hydraulic conductivity for two-dimensional flow is obtained as:

$$q_x = -K_{xx}\frac{\partial\phi}{\partial x} - K_{xy}\frac{\partial\phi}{\partial y}$$
$$q_y = -K_{yx}\frac{\partial\phi}{\partial x} - K_{yy}\frac{\partial\phi}{\partial y} \quad (3.6)$$

where,

$$K_{xx} = K_1 \cos^2\alpha + K_2 \sin^2\alpha$$
$$K_{xy} = K_{yx} = (K_1 - K_2) \sin\alpha \cos\alpha \qquad (3.7)$$
$$K_{yy} = K_1 \sin^2\alpha + K_2 \cos^2\alpha$$

For the general case of three-dimensional flow, the Darcy's law is given by:

$$q_x = -K_{xx}\frac{\partial\phi}{\partial x} - K_{xy}\frac{\partial\phi}{\partial y} - K_{xz}\frac{\partial\phi}{\partial z}$$
$$q_y = -K_{yx}\frac{\partial\phi}{\partial x} - K_{yy}\frac{\partial\phi}{\partial y} - K_{yz}\frac{\partial\phi}{\partial z} \qquad (3.8)$$
$$q_z = -K_{zx}\frac{\partial\phi}{\partial x} - K_{zy}\frac{\partial\phi}{\partial y} - K_{zz}\frac{\partial\phi}{\partial z}$$

The nine coefficients K_{ij} (i =x,y,z; j = x,y,z) are known as the coefficients of the hydraulic conductivity tensor given by:

$$\bar{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \qquad (3.9)$$

which is a symmetric matrix with the diagonal elements K_{xx}, K_{yy} and K_{zz}.

3.1.1 HYDRAULIC CONDUCTIVITY ELLIPSE

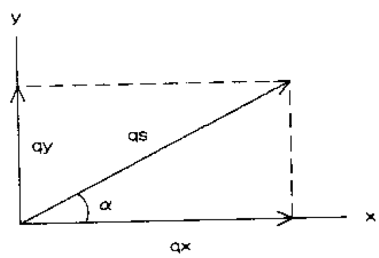
Consider an arbitrary flowline in the xy-plane in a homogeneous, anisotropic medium with principal hydraulic conductivities K_x and K_y (Fig. 3.2a). Along the flow line

$$q_s = -K_s \frac{\partial\phi}{\partial s} \qquad (3.10)$$

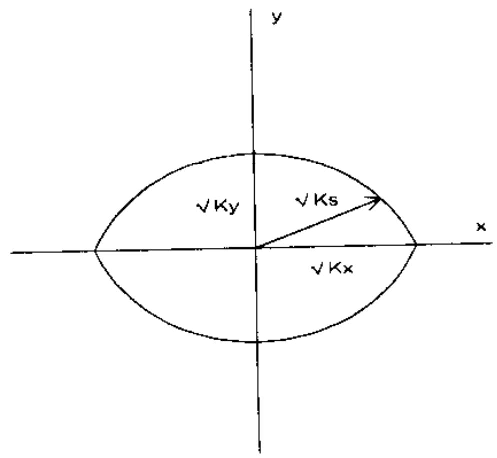
where K_s is the hydraulic conductivity in the direction of q_s. Though it is unknown, it should lie within the range of principal hydraulic conductivity values $[K_x, K_y]$. Resolving q_s into its components, q_x and q_y we get:

$$q_x = -K_x \frac{\partial \phi}{\partial x} = q_s \cos\alpha \qquad (3.11)$$

$$q_y = -K_y \frac{\partial \phi}{\partial y} = q_s \sin\alpha$$



[a]



[b]

Fig. 3.2 [*a*] Specific discharge q_s in an arbitrary direction of flow, and [*b*] the hydraulic conductivity ellipse (after Freeze and Cherry, 1979)

Now, the potential (head) in the aquifer is a function of space, $\phi = \phi(x,y)$. Therefore:

$$\frac{\partial h}{\partial s} = \frac{\partial h}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial h}{\partial y} \cdot \frac{\partial y}{\partial s} \qquad (3.12)$$

But, geometrically, $\partial x/\partial s = \cos\alpha$ and $\partial y/\partial s = \sin\alpha$. Substituting these relationships together with the above equations and simplifying yields:

$$\frac{1}{K_s} = \frac{\cos^2\alpha}{K_x} + \frac{\sin^2\alpha}{K_y} \tag{3.13}$$

Eqn.(3.13) relates the principal conductivity components K_x and K_y to the resultant K_s in any angular direction, α. The corresponding equation in rectangular coordinates may be obtained, by putting $x = r\cos\alpha$ and $y = r\sin\alpha$, as:

$$\frac{r^2}{K_s} = \frac{x^2}{K_x} + \frac{y^2}{K_y} \tag{3.14}$$

Eqn.(3.14) reprents an ellipse with major axes $\sqrt{K_x}$ and $\sqrt{K_y}$ (Fig. 3.2b) and it is known as the hydraulic conductivity ellipse. Thus, it is possible to determine the hydraulic conductivity value K_s for any direction of flow in an anisotropic medium. From the above results it can be deduced as a corollary that the direction of the stream lines will not coincide with the direction of the normal to the equipotential lines.

3.2 COEFFICIENT OF ANISOTROPY

Examining the components of specific discharge vector [*eqn.(3.3)*], it is clear that we can not define a single potential ϕ since the coefficients in q^*_x and q^*_y are different. However, it is possible to transform the flow domain to a different domain with with coordinates, say X and Y such that a potential ϕ may be defined in that domain. Let the transformation be expressed as:

$$\begin{aligned} X &= x^* \\ Y &= \beta y^* \end{aligned} \tag{3.15}$$

It is possible to choose β such that the hydraulic potential ϕ satisfies Laplace's equation in the transformed domain.

We have the continuity equation written in terms of x^*, y^* coordinate system as:

$$\frac{\partial q_x^*}{\partial x^*} + \frac{\partial q_y^*}{\partial y^*} = 0 \tag{3.16}$$

Substitution of eqn.(3.3) in to the equation of continuity provides:

$$-K_1 \frac{\partial^2 \phi}{\partial (x^*)^2} - K_2 \frac{\partial^2 \phi}{\partial (y^*)^2} = 0 \tag{3.17}$$

Applying the transformation in eqn.(3.15) yields:

$$-K_1 \frac{\partial^2 \phi}{\partial X^2} - K_2 \beta^2 \frac{\partial^2 \phi}{\partial Y^2} = 0 \tag{3.18}$$

By choosing $\beta = \sqrt{(K_1 / K_2)}$, it can be seen that the potential ϕ satisfies the Laplace's equation in terms of X and Y. Hence, the potential can be evaluated in the transformed domain. β, the root of the ratio of hydraulic conductivities along the principal directions, is termed as the *coefficient of anisotropy*.

However, the equivalent isotropic hydraulic conductivity, K_{IS} in the transformed domain is needed to compute flow rates from solution. By virtue of a simple flow-net analysis it can be shown that the equivalent isotropic hydraulic conductivity,$K_{IS} = \sqrt{(K_1 K_2)}$.

Now, a solution of eqn.(3.18) for ϕ can be obtained in terms of the coefficient of anisotropy and the equivalent isotropic hydraulic conductivity as:

$$\phi\,(X, Y) = \frac{1}{2\pi K_{IS}\sqrt{X^2 + \beta^2 Y^2}} \tag{3.19}$$

Eqn.(3.19) is valid for an aquifer system in which the orientation of soil strata is horizontal. For a general case when the bedding planes of the strata are inclined to the horizontal, an appropriate transformation of coordinates can be performed to obtain the potentials. It will be seen that the equipotential lines of ϕ will be following elliptical paths in the vertical section of the aquifer system.

3.3 LAYERED HETEROGENEITY AND ANISOTROPY

Sedimentary soils often consists of thin alternating layers of varying permeabilities as a result of particle oreintation. It can be shown that a stratified medium of homogeneous and isotropic layers can be converted into an equivalent single homogeneous and anisotropic layer.

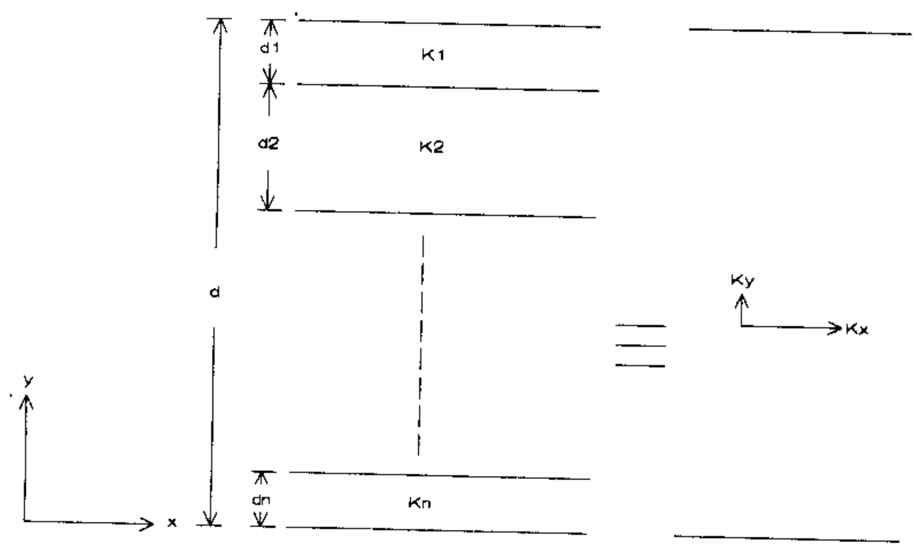


Fig. 3.3 A stratified-heterogeneous aquifer and its equivalent single-homogeneous-anisotropic aquifer (after Freeze and Cherry, 1979)

Consider a layered soil formation with varying permeabilities for the layers as shown in Figure-3.3. Eeach layer is homogeneous and isotropic with respective hydraulic conductivity values $K_1, K_2, \ldots, K_n$ and thicknesses $d_1, d_2, \ldots, d_n$. Let us consider flow perpendicular to the layering. By continuity, the specific discharge, q must be the same entering the system as it is leaving and it should be constant throughout the system. Let $\Delta\phi_1$ be the head-loss across the first layer, $\Delta\phi_2$ the head-loss across the second layer, and so on.

Then, the total head-loss will be:

$$\Delta\phi = \Delta\phi_1 + \Delta\phi_2 + \ldots + \Delta\phi_n \tag{3.20}$$

By Darcy's law, specific discharge,

$$q = K_1 \frac{\Delta\phi_1}{d_1} = K_2 \frac{\Delta\phi_2}{d_2} = \ldots = K_n \frac{\Delta\phi_n}{d_n} = K_y \frac{\Delta\phi}{d} \tag{3.21}$$

where K_y is the equivalent vertical hydraulic conductivity for the layered system. From eqn.(3.21) as well as eqn.(3.20):

$$K_y = \frac{qd}{\Delta\phi} = \frac{qd}{\Delta\phi_1 + \Delta\phi_2 + \ldots + \Delta\phi_n} = \frac{qd}{qd_1/K_1 + qd_2/K_2 + \ldots qd_n/K_n} \tag{3.22}$$

Therefore, an equivalent vertical hydraulic conductivity for a layered system is related by the actual hydraulic conductivities and thicknesses of individual layers and is given by:

$$K_y = \frac{d}{\sum_{i=1}^{n} d_i/K_i} \tag{3.23}$$

Now, consider flow parallel to the layering. Let $\Delta\phi$ be the head-loss over a horizontal distance, L. The discharge Q from the system is the sum of the discharges through the layers. The specific discharge, q = Q/d is therefore given by,

$$q = \sum_{i=1}^{n} \frac{K_i d_i}{d} \cdot \frac{\Delta\phi}{L} = K_x \frac{\Delta\phi}{L} \tag{3.24}$$

where K_x is the equivalent horizontal hydraulicconductivity of the system and is given by:

$$K_x = \sum_{i=1}^{n} \frac{K_i d_i}{d} \tag{3.25}$$

Eqn.(3.25) and eqn.(3.23), respectively, provide the hydraulic conductivity values K_x and K_y (parallel and perpendicular to the stratification respectively) for a single-homogeneous-anisotropic formation which is hydraulically equivalent to a layered (heterogeneous) aquifer system of homogeneous-isotropic geological formations. Further, it can be shown using equations (3.23) and (3.25) that $K_x > K_y$ for all possible sets of values of K_i ($i=1,2,\ldots,n$)); in other words, the equivalent hydraulic conductivity in the direction of plane of stratification is greater than that perpendicular to the strata.

4.0 METHODOLOGY

Simulation of hydraulic potentials due to a point source of strength Q [L^3T^{-1}] placed at the surface of an aquifer system is carried out in the case of an (i) anisotropic, single-unconfined aquifer system, (ii) a layered-heterogeneous aquifer system, and (iii) stratified, heterogeneous and anisotroipic aquifer system.

An appropriate transformation of the anisotropic medium into an isotropic domain is carried out. The isotropic hydraulic potentials are then computed in the fictitious domain by one of the methods applicable for homogeneous isotropic aquifers. Finally the fictitious hydraulic potentials are transferred back to the actual domain yielding the required anisotropic hydraulic potentials.

4.1 TRANSFORMATION OF ANISOTROPIC FLOW DOMAIN

The analysis followed presents a methodology (*Strack, 1989*) for the transformation of anisotropic flow domain on to a fictitious isotropic domain for two-dimensional flow systems. The methodology has been adapted with some modifications to suit the present investigations.

We denote the Cartesian coordinates in the physical plane as (x^p,y^p), where the x^p and y^p axes are horizontal and vertical, respectively. The major principal direction of the hydraulic conductivity tensor makes an angle α with the x^p axis. Now, the Cartesian coordinates (x^{p*},y^{p*}) is chosen such that the x^{p*} axis is inclined at an angle α to the x^p axis (Fig. 4.1a). The Cartesian coordinates (x^{t*},y^{t*}) in the transformed domain (Fig. 4.1b), labeled by the superscript t, are chosen such that they correspond to the coordinates (x^{p*},y^{p*}) in the physical plane. Finally, the Cartesian coordinate system (x^t,y^t) is introduced in the transformed domain such that the x^t axis correspond to x^p axis.

The coordinates (x^{p*},y^{p*}) are expressed in terms of (x^p,y^p) by:

$$
\begin{aligned}
x^p &= x^{p*}\cos\alpha - y^{p*}\sin\alpha \\
y^p &= x^{p*}\sin\alpha + y^{p*}\cos\alpha
\end{aligned}
\qquad (4.1)
$$

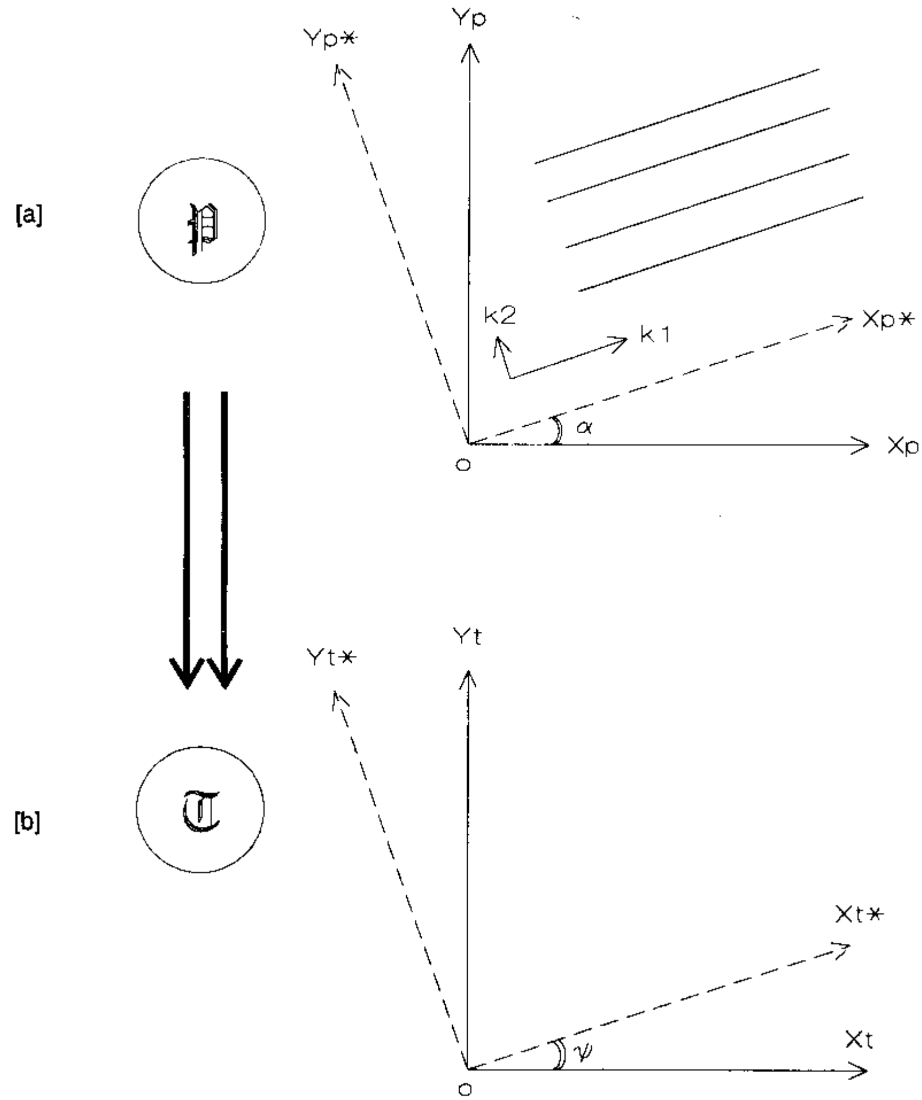


Fig. 4.1 Scheme of transformation of [a] the anisotropic physical plane onto [b] an isotropic domain; K_1, K_2 are the hydraulic conductivity values along the principal directions of anisotropy, and α is the angle of dip of the bedding planes.

The inverse of this transformation is given by:

$$\begin{aligned} x^{p*} &= x^p\cos\alpha + y^p\sin\alpha \\ y^{p*} &= -x^p\sin\alpha + y^p\cos\alpha \end{aligned} \qquad (4.2)$$

Using the following transformation into the isotropic domain

$$\begin{aligned} x^{t*} &= x^{p*} \\ y^{t*} &= \beta y^{p*} \end{aligned} \qquad (4.3)$$

where $\beta = \sqrt{(K_1/K_2)}$

If the angle between the x^{t*} and x^t axes is ψ, then

$$\begin{aligned} x^t &= x^{t*}\cos\psi - y^{t*}\sin\psi \\ y^t &= x^{t*}\sin\psi + y^{t*}\cos\psi \end{aligned} \qquad (4.4)$$

The x^t axis in the transformed domain corresponds to the x^p axis in the physical plane. Since $y^t = 0$ along the x^t axis, we have from eqn.(4.4):

$$\frac{y^{t*}}{x^{t*}} = -\tan\psi \; ; \quad for \;\; y^t = 0 \qquad (4.5)$$

Also, $y^p = 0$ along the x^p, so that from eqn.(4.1):

$$\frac{y^{p*}}{x^{p*}} = -\tan\psi \; ; \quad for \;\; y^p = 0 \qquad (4.6)$$

Combining eqn.(4.3), eqn.(4.5), and eqn.(4.6) provides an expression for ψ in terms of α as:

$$\tan\psi = \beta\tan\alpha \qquad (4.7)$$

It is now possible to express x^t and y^t in terms of x^p and y^p:

$$x^t = (x^p\cos\alpha + y^p\sin\alpha)\cos\psi - \beta(-x^p\sin\alpha + y^p\cos\alpha)\sin\psi$$
$$y^t = (x^p\cos\alpha + y^p\sin\alpha)\sin\psi + \beta(-x^p\sin\alpha + y^p\cos\alpha)\cos\psi \tag{4.8}$$

This may be re-written as:

$$x^t = \cos\alpha\cos\psi\,[\,x^p(1+\beta\tan\alpha\tan\psi) + y^p(\tan\alpha-\beta\tan\psi)\,]$$
$$y^t = \cos\alpha\cos\psi\,[\,x^p(\tan\psi-\beta\tan\alpha) + y^p(\tan\alpha\tan\psi+\beta)\,] \tag{4.9}$$

It is possible to select the coordinate transformations in such a way that $-\pi/2 \le \alpha \le \pi/2$. Further, from eqn.(4.7) it is evident that ψ also can be chosen such that $-\pi/2 \le \psi \le \pi/2$. Then without loss of generality the range of analysis can be restricted to the case where:

$$-\frac{\pi}{2} \le \alpha \le +\frac{\pi}{2}$$
$$-\frac{\pi}{2} \le \psi \le +\frac{\pi}{2} \tag{4.10}$$

Therefore,

$$\cos\psi \ge 0 \quad \forall \quad \psi \tag{4.11}$$

and we have,

$$\cos\psi = \frac{1}{\sqrt{1+\tan^2\psi}} \tag{4.12}$$

Using the expression for <u>$\cos\psi$</u> in eqn.(4.12) with eqn.(4.7) and eqn.(4.9) and simplifying gives:

$$x^t = x^p\sqrt{\cos^2\alpha+\beta^2\sin^2\alpha} + y^p\frac{(1-\beta^2)\sin\alpha\cos\alpha}{\sqrt{\cos^2\alpha+\beta^2\sin^2\alpha}} \tag{4.13}$$

$$y^{t} = y^{p} \frac{\beta}{\sqrt{\cos^{2}\alpha + \beta^{2}\sin^{2}\alpha}} \tag{4.14}$$

The expressions in eqn.(4.13) and eqn.(4.14) relates x^t and y^t of the transformed domain to the corresponding coordinate x^p and y^p in the original domain.

5.0 SIMULATION OF HYDRAULIC POTENTIALS

Simulation of hydraulic potentials is carried out in a cylindrically symmetrical unconfined anisotropic aquifer. Equipotential lines are drawn for the vertical section of the aquifer system. Referring to Fig. 3.1 (in section 3.0), the hypothetical aquifer system is formed by a number of strata with inclined bedding planes. The bedding planes of the soil layers make an angle α with the horizontal axis. The angle of dip (α) of the bedding planes with the horizontal is varied between zero and $\pi/2$ for various cases. The principal directions of anisotropy are along the bedding plane of the strata and perpendicular to it. The corresponding anisotropic hydraulic conductivities are K_1 and K_2 respectively. A point source of strength, Q[L^3T^{-1}] located at the centre of the system is maintaining the hydraulic potentials in the aquifer system by steady-state recharge from the top. The boundaries of the hypothetical aquifer system are assumed to be at very large distances from the source thereby extending the aquifer system to infinite distance. As such, at infinite distance from the source hydraulic potentials tend to be zero.

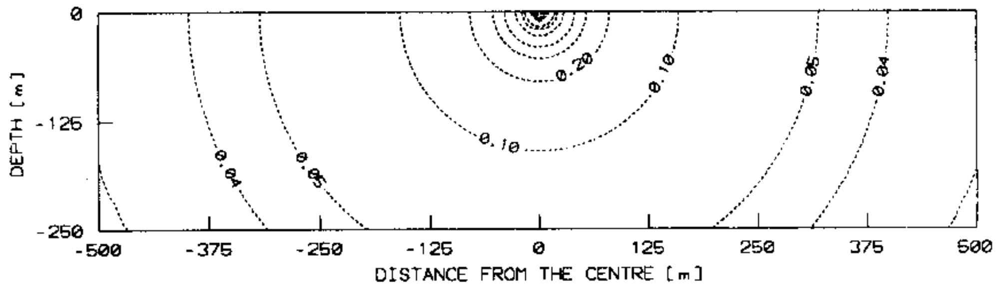
Simulation of hydraulic potentials are carried out for various levels of anisotropy in the aquifer and different orientations of the strata. Simulation of hydraulic potentials is carried out in the following types of aquifer systems:

(i) An anisotropic, single-unconfined aquifer with inclined strata

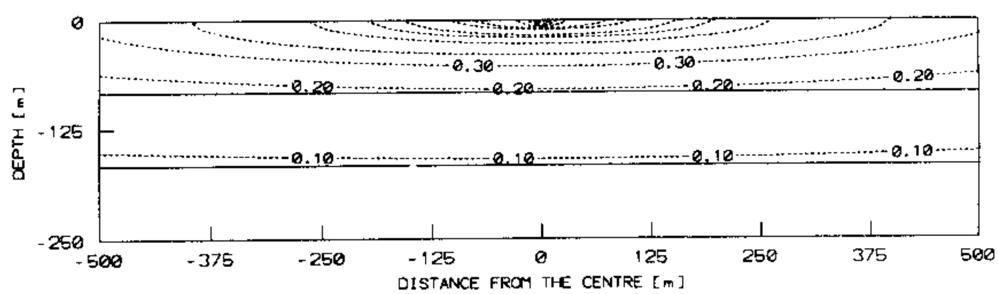
(ii) A horizontally-layered heterogeneous aquifer system where individual layers are isotropic

An algorithm ANISOT has been devised based on the transformation procedure detailed earlier (section 4.1) and used to simulate the hydraulic potentials in a vertical section of the aquifer system. Using input information on source strength (Q), Hydraulic conductivity values in the principal directions (K_1, K_2), number of layers (n), angle of dip of the strata (α), and grid (r,z) the algorithm computes the steady state hydraulic potentials in the anisotropic unconfined-infinite aquifer system.

Though it is assumed that the aquifer system extends to infinite distance, only a finite extent of the system is used for plotting the equipotential lines to allow reprographic convenience.



[a]



[b]

Fig. 5.1 Selected equipotential lines in: [a] an isotropic aquifer where inclination of bedding planes, $\alpha = 0$ and coefficient of anisotropy, $\beta = 1$; [b] in an anisotropic aquifer where inclination of bedding planes, $\alpha = 0$ and coefficient of anisotropy, $\beta = 10$

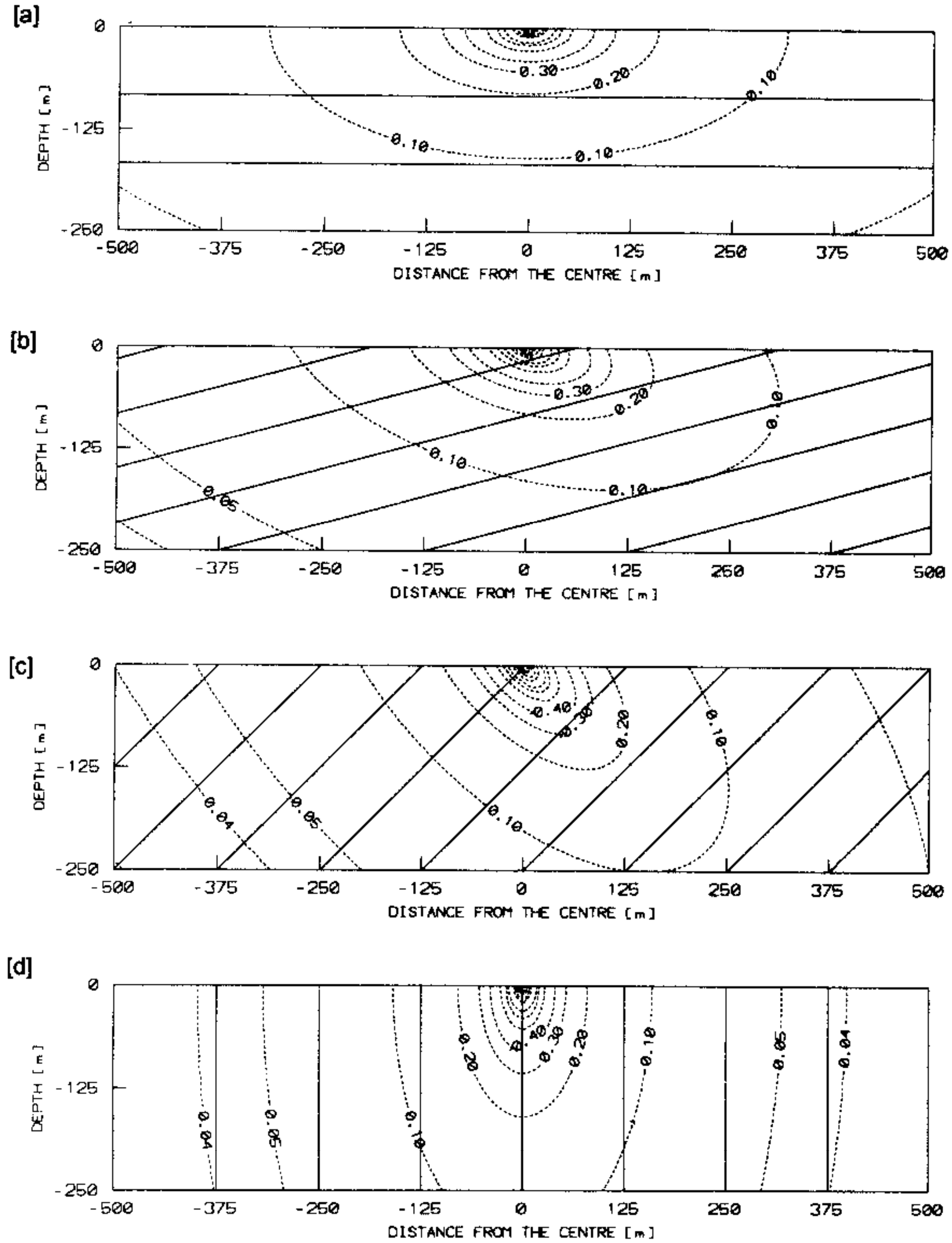


Fig. 5.2 Equi-potentials (dotted lines) in a stratified anisotropic aquifer system for different inclinations (α) of the bedding planes (solid lines) when the coefficient of anisotropy, β=2. [a] For α=0; [b] For $\alpha=\pi/12$; [c] For $\alpha=\pi/4$; [d] For $\alpha=\pi/2$.

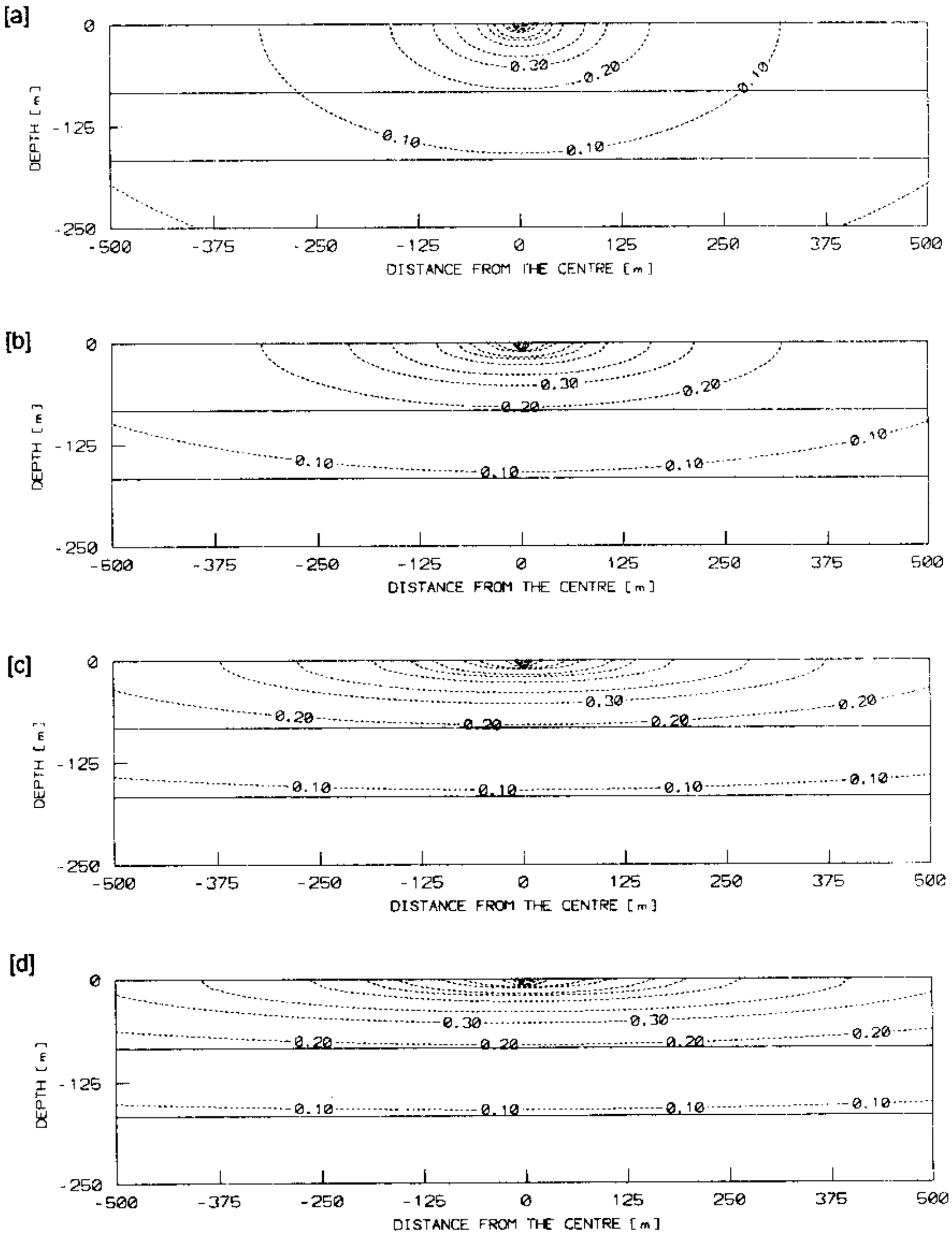


Fig. 5.3 Hydraulic potentials in an anisotropic aquifer for different coefficients of anisotropy (β) when the angle of dip of the strata, α=0. [a] For β=2; [b] For β=4; [c] For β=7; [d] For β=10.

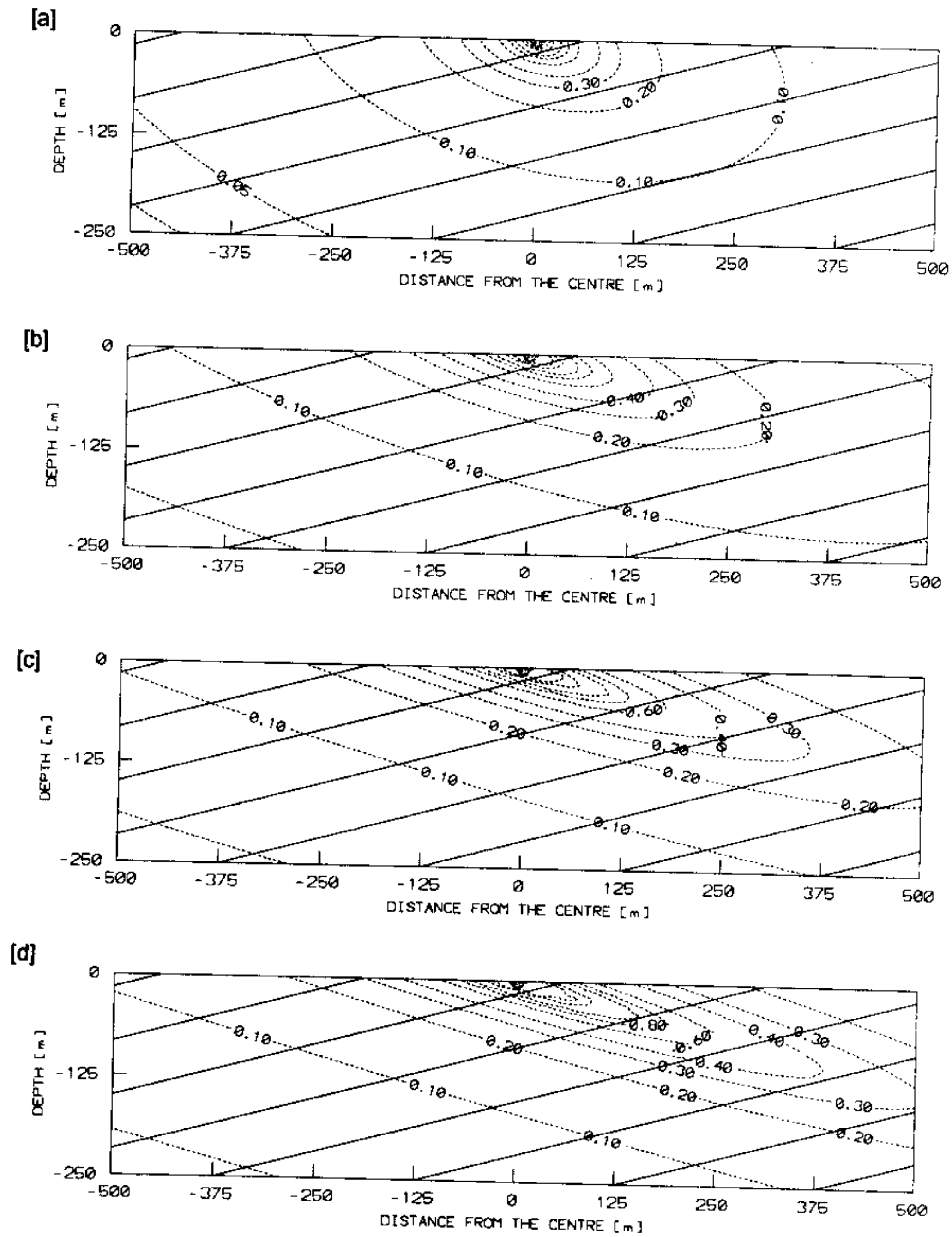


Fig. 5.4 Hydraulic potentials in an anisotropic aquifer for different coefficients of anisotropy (β) when the angle of dip of the strata, $\alpha=\pi/12$. [a] For β=2; [b] For β=4; [c] For β=7; [d] For β=10.

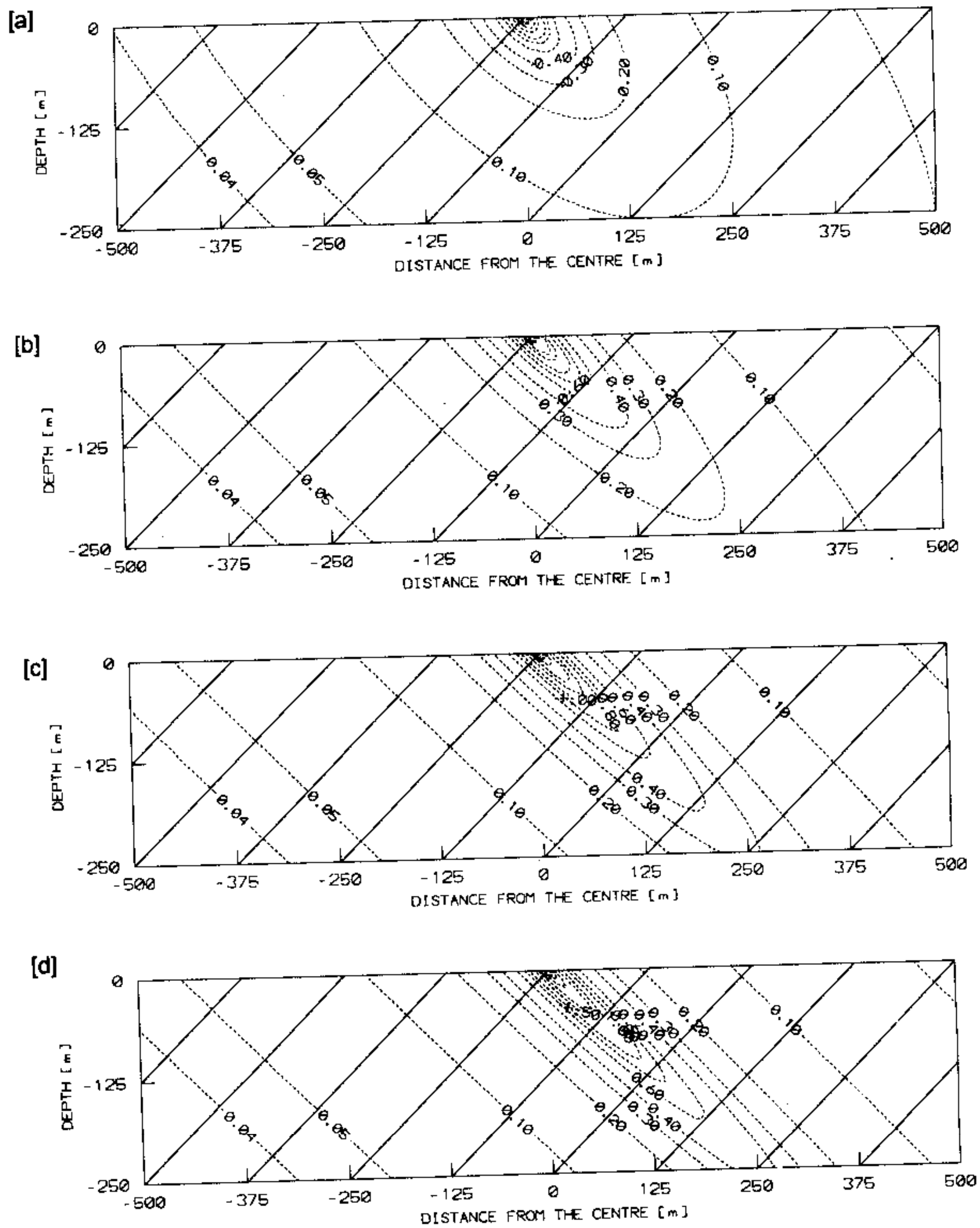


Fig. 5.5 Hydraulic potentials in an anisotropic aquifer for different coefficients of anisotropy (β) when the angle of dip of the strata, $\alpha=\pi/4$. [a] For $\beta=2$; [b] For $\beta=4$; [c] For $\beta=7$; [d] For $\beta=10$.

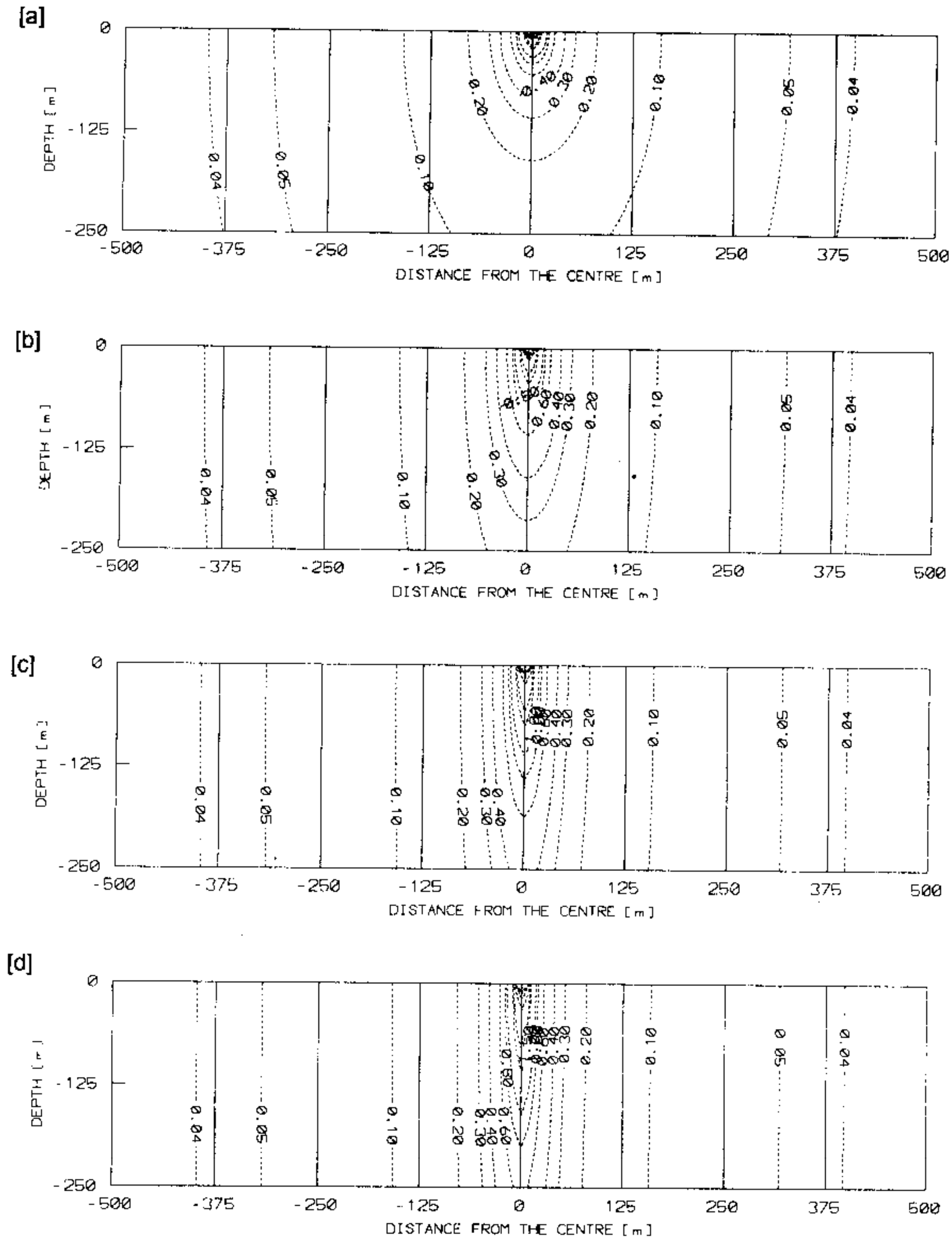
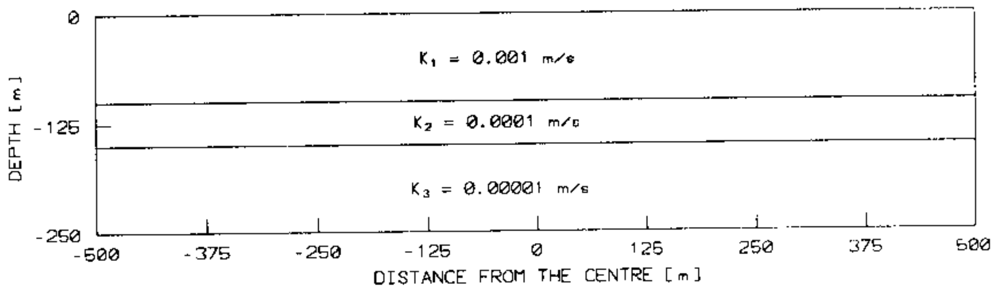
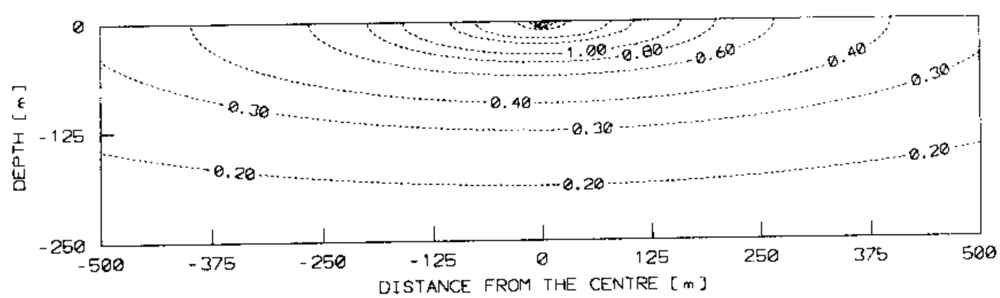


Fig. 5.6 Hydraulic potentials in an anisotropic aquifer for different coefficients of anisotropy (β) when the angle of dip of the strata, $\alpha=\pi/2$. [a] For $\beta=2$; [b] For $\beta=4$; [c] For $\beta=7$; [d] For $\beta=10$.



[a]



[b]

Fig. 5.7 A three-layered heterogeneous aquifer system (with isotropic layers) has been replaced by an equivalent single anisotropic system; [a] sketch of the actual layered aquifer system, [b] equi-potentials in the equivalent anisotropic system where α=0 and β=4.25 with K_1=0.000425 m/s.

5.1 ANISOTROPIC SINGLE-UNCONFINED AQUIFER

The set of cases with different coefficients of anisotropy and bedding plane inclinations chosen for simulating hydraulic potentials in the anisotropic, single-unconfined aquifer type are given in Table 5.1.1.

Sets	A- ANISOTROPIC SINGLE AQUIFER			
	$\alpha=0$	$\alpha=\pi/12$	$\alpha=\pi/4$	$\alpha=\pi/2$
$\beta=1$	✓	x	x	x
$\beta=2$	✓	✓	✓	✓
$\beta=4$	✓	✓	✓	✓
$\beta=7$	✓	✓	✓	✓
$\beta=10$	✓	✓	✓	✓

Table 5.1.1 Set of cases with different combinations angles of dip, α and coefficients of anisotropy, β

The aquifer parameters used in the simulations are given in Table 5.1.2:

PARAMETER	VALUE
Source strength, Q	0.1 m^3/s
Hydraulic conductivity in the major direction, K_1	0.001 m/s
Angle of dip of strata, α	0, $\pi/12$, $\pi/4$, $\pi/2^c$
Coefficient of anisotropy, β	1, 2, 4 7, 10
Ratio of hydraulic conductivity values, K_1/K_2	1, 4, 16, 49, 100
Lateral extent of the aquifer system simulated, L	1000 m
Depth of the aquifer system simulated, D	250 m

Table 5.1.2 Aquifer parameters used in the simulations for various set of cases

Fig. 5.1 compares the hydraulic potentials in an isotropic aquifer with that in an anisotropic aquifer where there is a horizontal stratification. The solid lines inside the plot indicate the stratification. The horizontal hydraulic conductivity, K_1=0.001 m/s in both the aquifers. In the case of isotropic aquifer (Fig.5.1a), as K_1 and K_2 are equal, the equipotentials form semi-circles around the source and radial-flow will be taking place uniformly in all directions. The horizontal hydraulic conductivity in the anisotropic aquifer is 100 times greater than that in the vertical direction. The shape of equipotentials in the anisotropic case (Fig.5.1b) is semi-elliptical clearly indicating the tendency of the flow to take place in the least resistive direction.

Fig. 5.2 depicts the hydraulic potentials in aquifer systems with different orientations of the strata. The thick lines in the plot indicate the orientation of the strata. The hydraulic conductivity values in the principal directions and coefficient of anisotropy are the same in all these case. Hence, the plots compare the distribution of hydraulic potentials in aquifer systems with varying inclinations of the soil strata with the horizontal.

Fig. 5.3 shows the equipotentials in four aquifer systems with different coefficients of anisotropy. The bedding planes of the strata are horizontal as indicated by the solid lines in the plot. The coefficient of anisotropy is varied from β=2 to β=10 for the cases (a), (b), (c), and (d) in Fig 5.3 respectively. The flattening of equipotential lines is evident as the system becomes more and more anisotropic. Same kind of plots as that of the previous one is presented in Fig. 5.4, Fig. 5.5, and Fig. 5.6 wherein the orientation of the strata are $\alpha = \pi/12$, $\alpha = \pi/4$, and $\alpha = \pi/2$ respectively for each case. These plots represent selected cases from the spectrum of all possible combinations of coefficient of anisotropy and orientation of bedding planes of soil strata in an anisotropic system. This helps one to visualise the pattern of distribution of heads in various anisotropic aquifer systems.

5.2 LAYERED-HETEROGENEOUS AQUIFER SYSTEM

It is discussed in section 3.3 that a layered heterogeneous aquifer system, where each individual layer is homogeneous and isotropic in itself (but, layer to layer variability exists), can be represented by an equivalent anisotropic aquifer. Though the hydraulic potentials in the original layered aquifer system and that in the equivalent anisotropic system differ, they will be equivalent as far as discharges are concerned. Therefore, the hydraulic potentials in such an equivalent aquifer can be simulated for assessing the flow from the layered system. In the present study one such layered aquifer system is chosen for simulating hydraulic potentials in its equivalent anisotropic aquifer.

Fig.5.7a is the sketch of the original layered aquifer system where the thicknesses of the top and the bottom layers are 100 m each while the middle layer is 50 m thick. The hydraulic conductivity of the top layer is 0.001 m/s and it reduces one order of magnitude each from top to bottom. Table 5.2.1 gives the aquifer parameters of a three-layered heterogeneous aquifer system used for the simulation of hydraulic potentials:

PARAMETERS	VALUES
Source strength, Q	0.1 m^3/s
Number of layers in the aquifer, n	3
Hydraulic conductivity of Layer-1, K_{TOP}	0.001 m/s
Hydraulic conductivity of Layer-2, K_{MID}	0.0001 m/s
Hydraulic conductivity of Layer-3, K_{BOT}	0.00001 m/s
Angle of dip of strata, α	0^c
Lateral extent of the aquifer system simulated, L	1000 m
Depth of the aquifer system simulated, D	250 m

Table 5.2.1 Aquifer parameters of the three-layered heterogeneous aquifer system used for the simulation of hydraulic potentials

Fig.5.7b shows the distribution of hydraulic potentials in the equivalent anisotropic aquifer of the three-layered heterogeneous aquifer system. The layered heterogeneous aquifer system has been simulated as an equivalent single-anisotropic aquifer with the parameters shown in Table 5.2.2:

PARAMETERS	VALUES
Hydraulic conductivity in the major direction, K_1	4.25 e-4
Hydraulic conductivity in the minor direction, K_2	2.36 e-5
Ratio of hydraulic conductivity values, K_1/K_2	18
Coefficient of anisotropy, β	4.25
Lateral extent of the aquifer system simulated, L	1000 m
Depth of the aquifer system simulated, D	250 m

Table 5.2.2 Parameters of the equivalent single-anisotropic aquifer of the three-layered heterogeneous aquifer system

Streamlines may be constructed for the equivalent anisotropic medium with appropriate techniques and the flow from the system can be assessed which would be the flow from the actual layered aquifer system.

6.0 SUMMARY AND CONCLUSION

The report provides a broad profile on the theory of anisotropic flow in porous media and reviews methodology for computing hydraulic potentials. A brief survey of the literature reveals that reported investigations on hydraulic potentials or flow in anisotropic aquifer systems are meagre. Further, simulation of hydraulic potentials in an unconfined anisotropic aquifer system, and also in an equivalent anisotropic system of a three-layered heterogeneous aquifer system due to a point source is carried out. An algorithm has been devised, using appropriate transformation technique and analytical results, to compute anisotropic hydraulic potentials. The hypothetical aquifer system is presumably formed by numerous soil strata inclined with the horizontal. The hydraulic conductivity values are assumed to be along the principal directions of anisotropy. Hydraulic potentials are simulated with varying coefficients of anisotropy and orientations of soil strata. The equipotential plots have been presented in the vertical section for visualising the pattern and behaviour of anisotropic hydraulic potentials in various cases.

It should be born in mind that the direction of flow and of the hydraulic gradient in an anisotropic porous medium are not parallel, generally. Hence, the angles between the directions of flow and of the hydraulic gradient are to be determined prior to constructing flow lines from equipotentials in an anisotropic aquifer system. By considering the hydraulic conductivity values in the principal directions, relationships can be established to determine the angles between the directions of flow and of the hydraulic gradient in the anisotropic domain (*Marcus, 1962*). Therefore, the present study can be augmented by incorporating an algorithm for the construction of flow lines too.

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