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ANALYSIS OF UNSTEADY FLOW FOR SUBSURFACE DRAINS

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PREFACE

Waterlogging can be caused by excess soil moisture due to periodic flooding, overflow by runoff, over irrigation, seepage, artesian water and impeded subsurface drainage. These conditions affect the growth and yield of crops and in course of time, turns the land saline or alkaline and ultimately render it unfit for cultivation. The valleys of Tigris and the Euphrates, which were once very fertile, were rendered barren because of this malady. Usually the cause of waterlogging in Indian subcontinent is limited to impeded drainage, over irrigation and inadequate drainage facilities. This is a very paradoxical situation where on one hand water being a scarce resource, is required to be conserved and its availability maintained through measures for maximising retention and minimising losses, and on the other hand indiscriminate use of water of limited availability results into waterlogging. In areas with periodic irrigations or high intensity rainfall, the assumption of a steady recharge is no longer justified. Under these conditions, unsteady state solution of the flow problem must be applied. A knowledge of the water table and its fluctuations in the irrigated area is therefore essential in assessing the possibilities of waterlogging. Even if any barrier is to be constructed to keep the polluted water one side and fresh water in another side in that case also a lateral drainage can serve the purpose. In view of the above, the present study has been carried out for the analysis of unsteady flow to subsurface drains.

This report entitled "Analysis of unsteady flow for subsurface drains" is the part of the research activities of 'Drainage Division' of the Institute. The study has been carried out by Dr. Vivekanand Singh, Scientist 'B' with the assistance of Shri S. L. Srivastava, R. A.

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ABSTRACT

In areas with periodic irrigations or high intensity rainfall, the assumption of a steady recharge is no longer justified. Under these conditions, unsteady state solution of the flow problem must be applied. A knowledge of the water table and its fluctuations in the irrigated area is therefore essential in assessing the possibilities of waterlogging. Even if any barrier is to be constructed to keep the polluted water one side and fresh water in another side in that case also a lateral drainage can serve the purpose. Keeping this in view, the analysis of unsteady flow to subsurface drains has been carried out.

In this study, a two-dimensional numerical model for subsurface flow has been developed for the analysis of unsteady flow for subsurface drains. The governing equation is the two-dimensional Richards equation in the mixed form. A strongly implicit finite-difference scheme has been used to solve the governing equation. The present model has been validated using the available analytical results for one side drain. The model has been used to simulate a hypothetical case of subsurface drains with parallel drains. The present numerical model can be used to simulate the unsteady subsurface drainage problem having one side drain and parallel drains (drain at both sides).

1.0 INTRODUCTION

Most of the ground water is located in the saturated zone, however, it is the unsaturated zone through which water recharges the saturated zone. If more irrigation water is applied to the field, more water will join to the ground water table and ultimately ground water table will rise. This ground water table may reach slowly to the root zone of the crop and finally affects the yield and field is called waterlogged.

An agricultural land is said to be waterlogged when the soil pores in the crop root zone gets saturated with water. This is usually caused by a rise of the subsoil water table. When the soil in the crop root zone becomes saturated, the plant roots are denied normal circulation of air, the level of oxygen declines and that of carbon-dioxide increases, as organic matter decomposes with the saturated results in wilting and ultimately in the death of the plants. An artificial subsurface drainage is provided in agricultural lands of in adequate natural drainage to guarantee suitable condition for plant growth.

Waterlogging can also be caused by excess soil moisture due to periodic flooding, overflow by runoff, over irrigation, seepage, artesian water and impeded subsurface drainage. These conditions affect the growth and yield of crops and in course of time, turns the land saline or alkaline and ultimately render it unfit for cultivation. The valleys of Tigris and the Euphrates, which were once very fertile, were rendered barren because of this malady. Usually the cause of waterlogging in Indian subcontinent is limited to impeded drainage, over irrigation and inadequate drainage facilities. This is a very paradoxical situation where on one hand water being a scarce resource, is required to be conserved and its availability maintained through measures for maximising retention and minimising losses, and on the other hand indiscriminate use of water of limited availability results into waterlogging.

In an arid region, the soils contain salts and when water evaporates, these are left on the surface as deposits. Rise of water table causes, in most cases, secondary soil salinization, either due to high salinity of the ground water or due to dissolution of solid phase salt by rising ground water. In such area when the capillary water comes to the surface, it increases solubility of harmful salts from the soil or those present in the ground water. This heavy concentration of salts renders the soil infertile.

The most effective answer to waterlogging is a properly designed drainage system. The drainage system of an area is the reverse of the irrigation system. Just as the main canal takes off from the river, branches off into distributaries and minors and finally ends in field channels supplying water to individual fields, in reverse order, the drainage is collected into main drains, and then discharged off in bigger stream.

1.1 Necessity of defining limits of waterlogging

The extent of waterlogging is measured by (i) visual survey of blocks of thur and salt appearance, (ii) determining depth of water table below ground, (iii) soil survey to deterioration of soil condition in different regions upto a depth of 3 m or so, (iv) geological survey to determine nature of ground water and location of aquifers and clay bands. In practice, when studies are done for localised small regions, all detailed surveys are carried out, however when general investigations are done to define the extent of waterlogging, usually the visual survey and measurement of depth of water table in wells are resorted to. There is no general agreement as to the depth of water table at which the land above it should be considered waterlogged. This system gives rise to varying figures of the waterlogged area, depending upon the definition used by the investigating agency in defining waterlogging. This would be clear by some case histories given subsequently.

1.2 Water-Logged Area in India due to irrigation

In India the first survey of waterlogged area was done in 1972 by the Irrigation Commission. They collected data through questionnaires and the desirable limit of water table was mentioned as 1.5 m below ground level.

The National Commission of Agriculture (1976) did not propose any such limit of water table depth and the figures for waterlogged land worked out by them was 6 million ha. The Commission also estimated that alkaline and saline soils together accounted for an area of 77 million ha.

Various other investigators gave varying figures, depending upon the choice of criteria of waterlogging adopted by them. The different findings are chartered in Table 1. Casual abtitute is taken by different States in giving such figures. Some States repeated the old figures taken decades ago and some increase the figures on the basis of actual or guessed data.

It can be seen that different agencies, all without proper and precise measurement, have given different figures (varying upto 600 %) for waterlogged land in India. Thus due to lack of uniformity of the criteria for categorising an area as waterlogged in a particular agro-climatic condition, the statistics in this regard are questionable in magnitude and in relevance.

Table 1: **Waterlogged Area in India According to Different Studies**

Sl	Year	Agency undertaking study	Water logged area in Mha	Water logged area in %	Percentage of	
					Mean Figure for WLA	Lowest Figure forWLA
1	1972	Irrigation Commission	4.84	12.2	56.1	141.4
2	1976	National Commission on Agriculture	5.986	13.2	69.3	17.3
3	1982	Central Ground Water Board	3.423	6.1	39.6	100.0
4	1984	Administrative Staff College of India	10.0	16.9	115.8	292.1
5	1989	G P Bhargava-Research Scholar	9.826	14.2	113.8	287.1
6	1989	World Watch Paper No. 3	20.0	28.9	231.7	584.3
7	1990	Ministry of Agriculture	8.526	12.3	98.8	249.1

Mha - Million Hectare; WLA - Water Logged Area

Secondly, quite often, the cited statics are meant for the entire land area suffering from waterlogging and salinity, without any distinction of the area affected due to canals and due to other causes like inundation of low lying areas, encroachment choking up of the natural drainage conditions, etc. In such circumstances, it is essential to go for uniform criteria to determine the waterlogged area, if we want to avoid wide as well as frequent variations in the extent of waterlogged areas in different regions. This could be done only if definitions are developed on the basis of agro-climatic zones.

In areas with periodic irrigations or high intensity rainfall, the assumption of a steady recharge is no longer justified. Under these conditions, unsteady state solution of the flow problem must be applied. A knowledge of the water table and its fluctuations in the irrigated area is therefore essential in assessing the possibilities of waterlogging. Even if any barrier is to be constructed to keep the polluted water one side and fresh water in another side in that case also a lateral drainage can serve the purpose. Keeping this in view, the analysis of unsteady flow to subsurface drains has been carried out.

Many authors have been analysed the drainage problems considering one-dimensional vertical flow. Two-dimensional numerical model has been developed for the saturated-unsaturted zone by Skaggs and Tang 1976, Vauclin et al. 1979 and Merva et al. 1983.
It is found that the problem of transient recharge of a water table aquifer can be solved correctly only by considering the flows in the entire saturated-unsaturated domain. Vauclin et al., 1979 shows that the classical saturated flow approach is totally unable to determine the transfer time for water in the unsaturated zone. Two-dimensional numerical model in the saturated zone has been developed by Ahmad et al., 1991. A composite flow model has been developed by Ahmad et al., 1993. In this model, two-dimensional ground water flow equation has been used for saturated zone and two-dimensional Richards equation in pressure head form has been used for unsaturated zone.

In this report, two-dimensional mixed form of Richards equation has been solved for the unsteady flow in the subsurface soil. A strongly implicit finite-difference scheme has been used for the solution of the mixed form of the Richards equation. Model has been validated with the available analytical results in the literature. A hypothetical case study has been carried out to show the applicability of the model after putting the drainage at some level below root zone at both sides.

2.0 GOVERNING EQUATIONS

Mathematical modelling for unsteady flow for subsurface drains involves the solution of the equation governing the flow. In this study, the subsurface flow is represented by the two-dimensional Richards equation in the mixed based form in x and z directions.

2.1 Richards Equation

The subsurface flow is considered as two-dimensional motion of a single-phase incompressible fluid. The two-dimensional, transient unsaturated flow equation in an isotropic porous medium is derived by applying the principle of conservation of mass and the basic Darcy's law for unsaturated flow and making the following assumptions.

(i) Compressibility of the medium and the water are negligible;

(ii) The air phase is stagnant and is at atmospheric pressure;

The two-dimensional continuity equation without sources and sinks within the flow domain for saturated and unsaturated case can be given as (Freeze and Cherry 1979):

$$\frac{\partial \theta}{\partial t} + \frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial Z} = 0 \tag{1}$$

in which, θ=volumetric moisture content; V_x and V_z = Darcy flow velocities in the x and z directions, respectively; and x and z are distances along the two coordinate axis in horizontal and vertical respectively. z is taken positive down wards. It is assumed that the Darcy's law is applicable for evaluating the velocity components. The Darcy's law for unsaturated flow in the x and z directions in an isotropic soil is

$$V_x = -K(\psi)\frac{\partial \psi}{\partial x}, \qquad V_z = -K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right) \tag{2}$$

in which, ψ = pressure (suction) head (m); and $K(\psi)$= unsaturated hydraulic conductivity (m/s); which depends on the pressure head, ψ. Substitution of Equation (2) in Equation (1) yields the Richards equation (Freeze and Cherry 1979):

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[K(\psi)\frac{\partial \psi}{\partial x}\right] + \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} - 1\right)\right] \tag{3}$$

Equation (3) is said to be in "mixed form" since it includes both the dependent variables θ and ψ. Most of the earlier studies on unsaturated ground water flow have employed the

Richards equation in either pressure head form or moisture content form. The pressure head form of the Richards equation is applicable to the flow in saturated and unsaturated zones and layered zones, but gives large mass balance errors. The moisture content form of the Richards equation perfectly conserves the mass within the flow domain, but is not applicable to the saturated flow zones and is not directly applicable to the layered zones because of the discontinuities in moisture content profiles at the interface of layers. These difficulties are over come by using the mixed form of Richards equation. The mixed form of Richards equation results better numerical behaviours then the other forms (Allen and Murphy (1986), Hill et al. (1989) and Celia et al. (1990)). It combines the benefit of both the pressure head and moisture content forms of Richards equation. The numerical models based on the mixed form of Richards equation can guarantee mass balance while having no limitations when applied to field problems (Singh, 1997). The main difficulty in the Richards equation to actual field situations is the estimation of the parameters of the soil characteristic curves. Characteristics relationships between the pressure head, ψ and the hydraulic conductivity, K (ψ-K relationship) and between the moisture content, θ and the pressure head, ψ (ψ-θ relationship) are needed while solving Eqs. (1) and (2) in the unsaturated zone. In general, ψ-K and ψ-θ relationships are not unique and soils exhibit different behaviour during wetting and drying phases. This hysteresis in soil characteristics is not considered for the cases studied in the present work. However, the hysteresis can be included by employing different ψ-K and ψ-θ relationships for wetting and drying processes. Several quasi-analytical equations are available to describe ψ-K and ψ-θ relationships in Rawls and Brakensick, 1988.

3.0 NUMERICAL SOLUTION

The two-dimensional Richards equation, Eq. 1, for subsurface flow have to be solved along with an appropriate boundary condition. In the present study, a recently developed strongly implicit finite-difference scheme (Hong et al. 1994 and Singh, 1997)) for the mixed based formulation of the Richards equation is used to simulate the unsaturated subsurface flow conditions. This scheme ensures mass balance in its solution regardless of time step size and nodal spacings, and has no limitations when applied to field problems (Celia et. al, 1990). It is also easy to incorporate different types of boundary conditions in this scheme.

Numerical solution of the Richards equation is described in the following section. The subsurface flow domain is divided into a number of rectangular blocks as shown in Fig. 1.

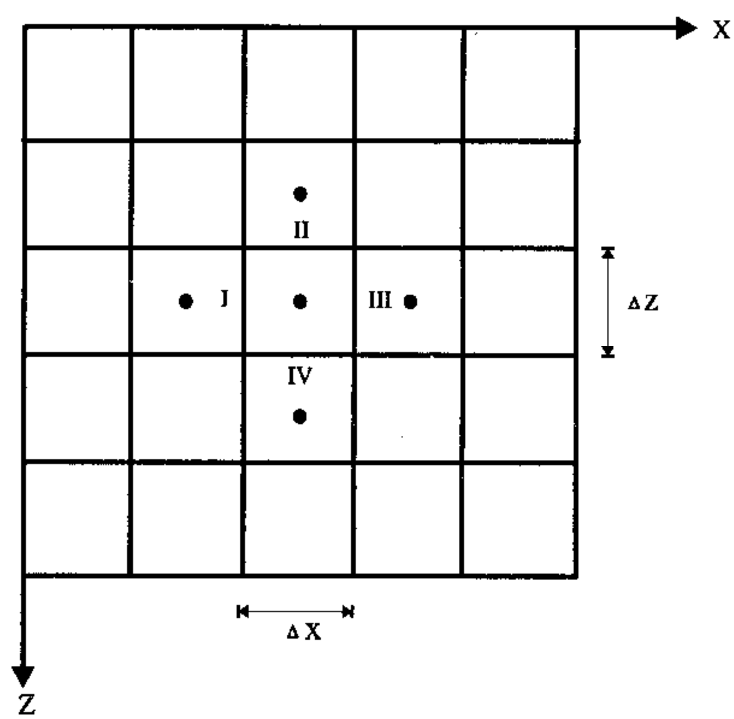

Fig. 1 Finite-Difference Grid

The moisture content, θ and the pressure head, ψ are specified at the centre of the block (node), while the velocities are specified at the interblock faces. The subscript i refers to the block number in the x-direction and the subscript j refers to the block number in the z-direction. The superscripts n and n+1 refer to the known and the unknown time levels, respectively. The finite-difference form of the Eq. (1) is

$$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{\Delta t} + \frac{\overline{V}_{III} - \overline{V}_{I}}{\Delta x} + \frac{\overline{V}_{IV} - \overline{V}_{II}}{\Delta z} = 0 \tag{4}$$

where, the bar is used to denote the time averaged value of the velocity. Δx and Δz are the nodal spacings in the x and z directions, respectively. The time-averaged velocities are determined by

$$\overline{V} = w V^{n+1} + (1-w) V^{n} \tag{5}$$

in which, w = time weighting factor. w = 1.0 for fully implicit scheme and it is equal to 0.5 for the Crank-Nicolson scheme. The velocity at any interblock face is determined using the pressure heads at the neighbouring cell centres. For example ;

$$V_{IV} = -\frac{K_{IV}\left\{(\psi_{i,j+1} - \psi_{i,j}) - \Delta z\right\}}{\Delta z} \tag{6}$$

and

$$V_{I} = -\frac{K_{I}\left(\psi_{i,j} - \psi_{i-1,j}\right)}{\Delta x} \tag{7}$$

in which, K_{IV} and K_I are the unsaturated hydraulic conductivities evaluated at the interblock faces IV and I, respectively. Substitution of Eqs. (5), (6) and (7) in Eq. (4) yields

$$\begin{aligned} Res_{i,j}^{n+1} = {} & \frac{w\,\Delta t}{\Delta x^2}\left[-K_{III}^{n+1}\left(\psi_{i+1,j}^{n+1} - \psi_{i,j}^{n+1}\right) + K_{I}^{n+1}\left(\psi_{i,j}^{n+1} - \psi_{i-1,j}^{n+1}\right)\right] \\ & + \frac{w\,\Delta t}{\Delta z^2}\left[-K_{IV}^{n+1}\left(\psi_{i,j+1}^{n+1} - \psi_{i,j}^{n+1} - \Delta z\right) + K_{II}^{n+1}\left(\psi_{i,j}^{n+1} - \psi_{i,j-1}^{n+1} - \Delta z\right)\right] \\ & + \theta_{i,j}^{n+1} - \left[\theta_{i,j}^{n} - (1-w)\frac{\Delta t}{\Delta x}\left(V_{III}^{n} - V_{I}^{n}\right) - (1-w)\frac{\Delta t}{\Delta z}\left(V_{IV}^{n} - V_{II}^{n}\right)\right] = 0 \end{aligned} \tag{8}$$

The unsaturated hydraulic conductivity at an interblock face is estimated using the pressure heads at the neighbouring cell centres. Haverkamp and Vauclin (1979) state that the geometric mean is the best choice for estimating the interblock hydraulic conductivities. However, Hong et al. (1994) reported that the iterative solution of Eq. (8) fails to converge if the above procedure is adopted for estimating the K. This is especially true for infiltration into initially very dry soils. The geometric mean is strongly weighted towards the lower value and therefore, water can not drain easily if the soil is initially dry. This results in a non-physical build up of pressure. In this study, the interblock hydraulic conductivity is estimated by the weighted arithmetic mean.

For example,

$$K_{IV} = \gamma K(\psi_{i,j}) + (1-\gamma)K(\psi_{i,j+1}) \tag{9}$$

in which, γ is the weight coefficient. Hong et al. (1994) suggest a value of 0.5 for γ.

Equation (8) is written for all the blocks in the flow domain and this results in a set of simultaneous algebraic equations in the unknowns $\psi(i,j)^{n+1}$. These simultaneous equations are highly non-linear since θ^{n+1} and K^{n+1} are non-linear functions of ψ^{n+1}. In the present study, they are solved by using the Newton-Raphson technique.

$$Res_{i,j}^{n+1,r} + \frac{\partial Res_{i,j}^{n+1,r}}{\partial \psi_m} \delta\psi_m = 0 \tag{10}$$

in which, r is the previous iteration level and $\delta\psi = (\psi^{n+1,r+1} - \psi^{n+1,r})$. Subscript m indicates the summation of the second term over all the blocks. Substituting Eq. (8) in Eq. (10) yields a linear equation in $\delta\psi$ having the following form.

$$\begin{aligned} &W_{i,j}^{n+1,r}\delta\psi_{i-1,j} + E_{i,j}^{n+1,r}\delta\psi_{i+1,j} + T_{i,j}^{n+1,r}\delta\psi_{i,j-1} \\ &+ B_{i,j}^{n+1,r}\delta\psi_{i,j+1} + P_{i,j}^{n+1,r}\delta\psi_{i,j} + Res_{i,j}^{n+1,r} = 0 \end{aligned} \tag{11}$$

in which, W, E, T, B, and P are the elements of the Jacobian of the system of equations, Eq. (8). Equations for evaluating these are presented in Appendix-I. Equation (11) when written for all the blocks in the domain constitutes a matrix equation

$$A^{n+1,r}\delta\psi = -Res^{n+1,r} \tag{12}$$

in which, the coefficient matrix A is banded. Equation (12) is solved in the present study using an efficient NAG LIBRARY, subroutine D03EBF, especially designed for systems such as this.

For convergence in iteration of Eq. (10), it is required that

$$\left|Res_{i,j}^{n+1,r}\right| < \epsilon \tag{13}$$

in which, ϵ = water content convergence tolerance. Equation (13) practically represents the principle of mass conservation because usually a very small value of ϵ in imposed.

3.1 Boundary Conditions:

(i) *Flux-Type Boundary Conditions*: In the present scheme, the grid is arranged in such a manner that the boundaries of the flow domain coincide with the top cell face. Therefore, flux or velocity-type boundary condition can be incorporated in a natural way in Eq. (4).

(ii) ***Pressure Head-Type Boundary condition***: Referring Fig. 2, let ψ_b be the imposed pressure head at the ground surface of the flow domain. This pressure head ψ_b is used along with the values of ψ_1 , pressure head at the first node and ψ_2, pressure head at the second node in the z-direction to determine the flux at the ground surface as given below

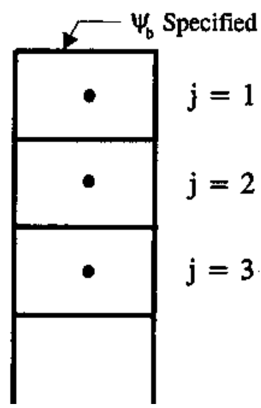

Fig. 2 Definition Sketch of Pressure Head type Boundary Condition

$$V_z\Big|_{z=0} = -K(\psi_b)\left[\frac{(-8\,\psi_b+9\,\psi_1-\psi_2)}{3\,\Delta z}-1\right] \tag{14}$$

Second-order forward finite-difference analog is used to determine the above Eq. (14). Equation (8) and equations for the coefficients W, E, T, B & P are appropriately changed to include the boundary conditions before the matrix $A^{n+1,r}$ in Eq.(12) is formed.

3.2 Boundary Conditions for seepage face:

For the unsteady flow condition, the length of the seepage face varies along with the time, in such manner that location of the seepage face is not known a priori but it must be calculated along with the solution. Two boundary conditions must satisfied along the seepage face. (i) If a zero-pressure head ($\Psi_s = 0$) is imposed at the boundary, the computed normal flux V_s at that boundary must be positive. (ii) If a zero-normal flux ($V_s = 0$) is imposed at the boundary, the computed pressure head Ψ_s at the boundary must be negative (type-II boundary). The following iteration algorithm has been applied to incorporate the above boundary conditions:

(a) At the beginning of each time step, the boundary-condition type for each boundary block is assumed to be identical to that obtained at the end of the previous time level.

(b) At the end of each iteration, the following conditions will be checked for every boundary block:

- → For type-I boundary, if $V_s = -K_s(-3\Psi_N + \Psi_{N-1}/3)/\Delta x$ is negative, the boundary-condition type is switched to type-II, where K_s = saturated hydraulic conductivity and N is the number of blocks in the row being considered.
- → For type-II boundary, if $\Psi_s = (-9\Psi_N - \Psi_{N-1})/8$ is positive, the boundary-condition type is switched to type-I.

The expressions for V_s and Ψ_s have been derived with the aid of quadratic interpolations. If none of the boundary-condition types needs to be altered and the convergence condition for the whole flow domain is satisfied at certain iteration, the iteration is then terminated. Otherwise, new iteration is required.

The free water surface (pressure head = 0) has been calculated by interpolation based on opposite sign of the pressure head at two successive blocks in the vertical direction.

A no flux boundary condition is imposed at the top and the bottom of the flow domain.

3.3 Time Stepping Scheme

Following time stepping scheme has been applied in the present study. The computation is started with an initial time step Δt_0, which is arbitrary choosen provided the convergence can be achieved. For subsequent time steps, time step sizes are increased in following manner $\Delta t_{n+1} = \Delta t_n$ if $I_n \geq I_{man}$ or $\Delta t_{n+1} = \xi \Delta t_n$ if $I_n < I_{man}$, I_n = number of iteration required for the previous time step and I_{man} is the maximum number of iteration imposed. In our computations, $\xi = 1.2$ and $I_{man} = 8$ are imposed. Δt should not be too large it will decrease the accuracy.

4.0 RESULTS AND DISCUSSION

4.1 Validation of the Model

In order to validate the present model a problem, as shown in the Fig. 3, has been considered. This problem was previously analysed by Rubin (1968), who used an Alternate Direction Implicit (ADI) finite-difference method for the solution of the Ψ- based Richards equation. In this problem, the length of the flow domain is 30 cm and width of the flow domain is 30 cm. The initial and boundary conditions have been given in Fig. 3. The domain was initially in static equilibrium. The water level at the right face of the domain was lowered suddenly from 30 cm to 10 cm and then maintained for the whole computation time. The soil characteristic equations for the unsaturated hydraulic conductivity and moisture content are as follows:

$$K = K_s \frac{A}{A + |\psi|^m} \tag{15}$$

$$\begin{aligned} \theta = \ & 0.6009 - 0.05708 \ln(10.0 - \psi) \\ & + 0.0594 / \cosh(0.747 + 0.0415 \ \psi) \\ & - 0.0132 \exp(0.4055 - 0.20 \ \psi) \end{aligned} \tag{16}$$

where, K = unsaturated hydraulic conductivity; saturated hydraulic conductivity, K_s = $1.1575 \text{X} 10^{-5}$ cm/s; A = 400; m = 2; θ = moisture content; Ψ = suction head. The other computational parameters are: $\Delta z = \Delta x = 1$ cm; Tolerance $\epsilon = 1.0 \text{X} 10^{-5}$ and time-weighting factor w = 1. Time steps were generated by the time-stepping.

Figure 4 shows the comparison of water table position at t = 2 hr. with the earlier computed values of Rubin (1968). The curve with solid line are the computed results with the present model and dashed line are the results of the Rubin (1968). The water table position has been located based on Ψ values and it has been located at zero suction head or where the sign of suction head is changed. Figure shows the good agreement between these two results. It can be seen from the Fig. 4 that the position of water table at the right side face is slightly lower than the earlier computed results. It is due to the boundary condition applied at the seepage face. Figure 5 shows the comparison of the distribution of outflux at the right side face at the time of 2 hours. Figure 5 also shows the good agreement between earlier results and the results computed by the present model. Figure 5 shows that initially the computed outflux value from the present model are higher than the outflux of Rubin (1968). This may be due to the initial condition applied in the present model. In Fig 5, the distribution of the computed outflux at 20 cm from top is not matching with the earlier results. This is due to the node position of the present model. In the present model the exact nodes are at 19.5 cm and 20.5 cm. The level of the water table after applying the boundary is 10 cm, which is 20 cm from top. So the exact point at 20 cm is not in the present model and it is at 19.5 cm and 20.5 cm. This is why the computed outflux value at the point is not matching.

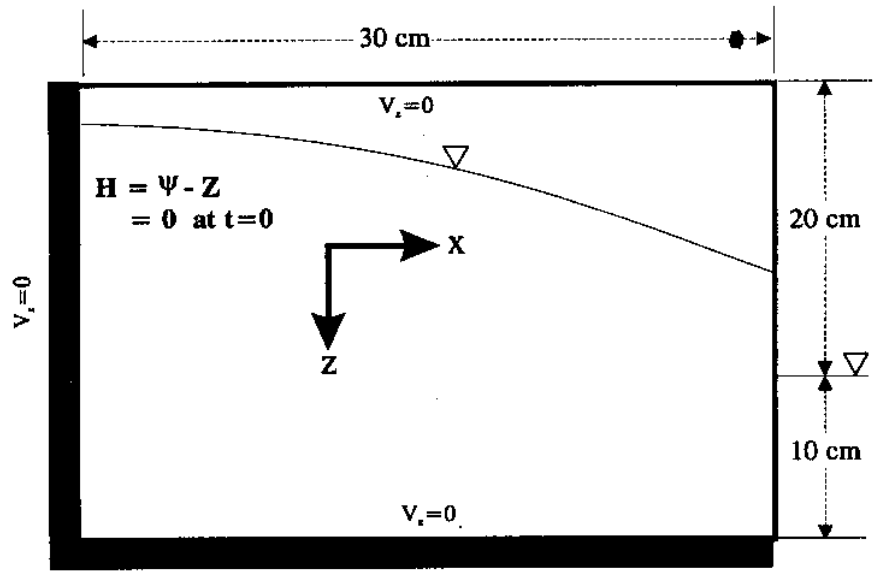

Fig. 3 Problem Definition Sketch for validation

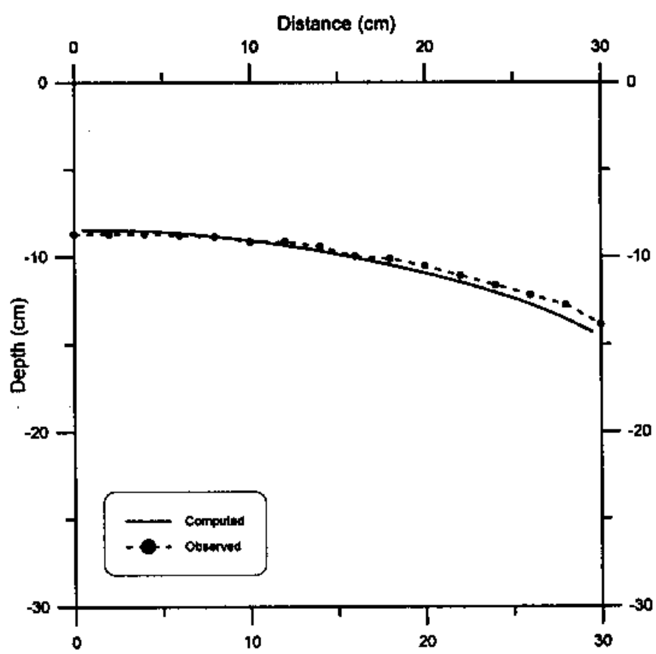

Fig. 4 Comparison of water table position at t=2 hrs.

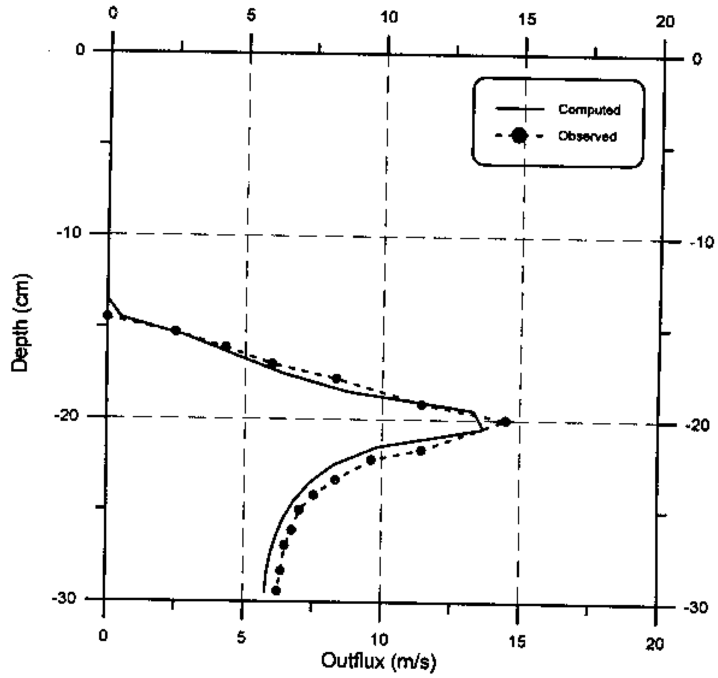

Fig. 5 Comparison of distribution of outflux at the right face at t = 2 hrs.

Figure 6 shows the contour of the computed hydraulic head at time of 2 hours. In this situation, initially the flow domain was fully saturated and at the end of the computation the minimum pressure head computed was -8.5 cm. This can be seen from the Fig. 6. Maximum pressure is at the bottom of the right face. As we know the direction of the flow under the ground takes place orthogonally to the lines of hydraulic head. From Fig. 6 it can be seen that the flow inside the saturated zone take place from left side to right side and below the seepage face and the stream lines will be very close near the exit face.

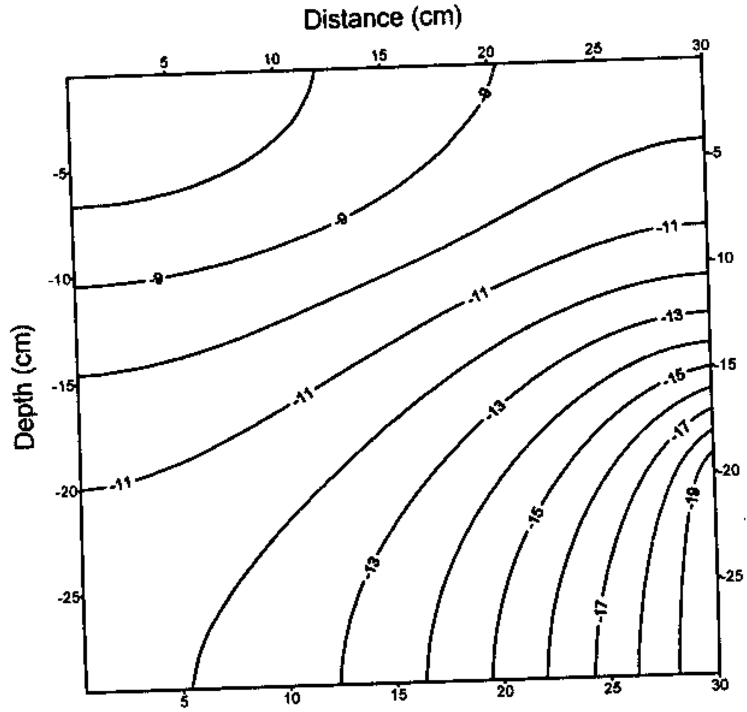

Fig. 6 Computed distribution of hydraulic head at t = 2 hrs.

4.2 Simulation of Hypothetical Case Study

A numerical simulation was carried out by using the present model to demonstrate the applicability of the model. For this purpose, a hypothetical domain as shown in Fig. 7 has been considered. The length and width of the domain are 50 cm and 30 cm respectively. Parallel drains have been considered both sides at a depth of 20 cm from the ground surface. All soil parameters and soil characteristics are taken same as in the previous problem except length and width. In this problem the seepage face has been considered both right and left side of the flow domain and water drain out form both sides as shown in Fig. 7

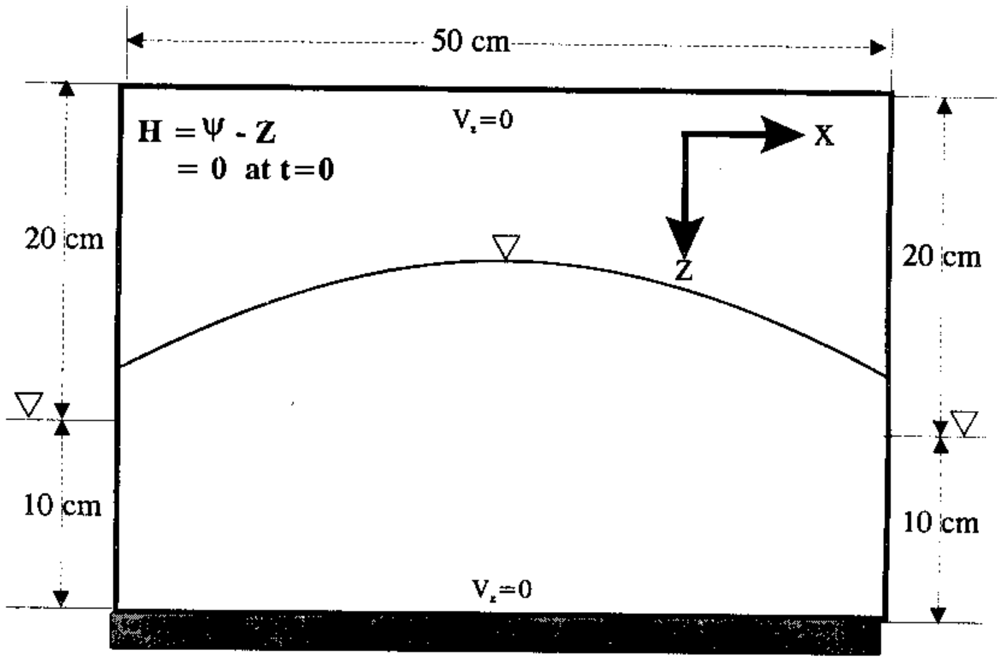

Fig. 7 Definition Sketch for the Simulation Problem (parallel drains)

Figure 8 shows the position of the water table at t = 2 hr. It is clear that as the time passes the position of the water table will go lower and lower till the drain level comes. At time t = 2 hrs. the position of the water table is as shown in Fig 8. From figure it can be seen that the water table position lowers at right and left both ends at a particular time. Figure 9 shows the distribution of computed outflux at the right face of the flow domain at the time of 2 hours. From Fig 8 it is clear that at t = 2 hrs the water table position (exit points) at both the end faces are at the drain levels i.e. close to the applied boundary condition. The outflux distribution also shows that computed outflux above the exit point is zero.

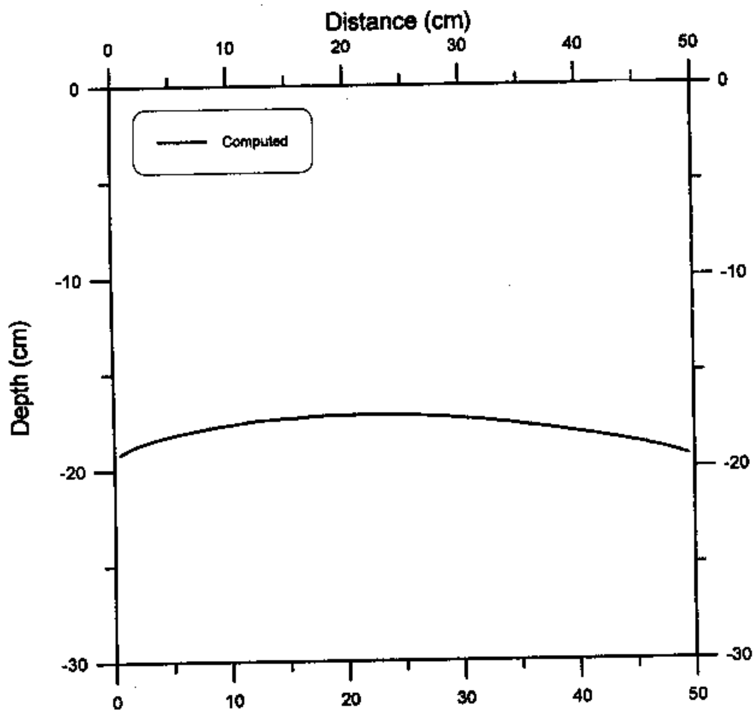

Fig. 8 Position of the water table at time, t, = 2 hrs.

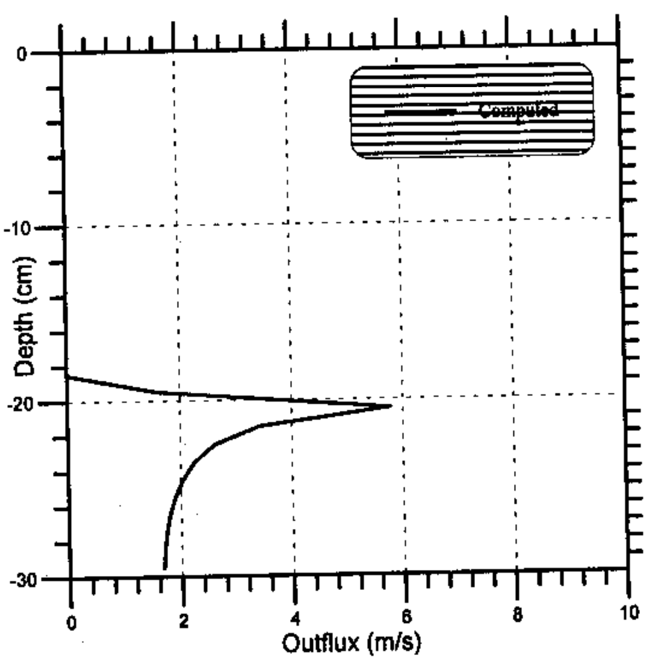

Fig. 9 Distribution of the computed outflux at the right face at time, t, = 2 hrs.

5.0 CONCLUSION

The numerical subsurface flow model has been developed using the two-dimensional mixed form of the Richards equation. The solution has been obtained using the strongly implicit finite-difference scheme. The present model has been validated using the available results for the one side drains. Validation shows the good agreement between both the results. The model has been used to simulate a hypothetical case of subsurface drains with parallel drains. The present numerical model can be used to simulate the unsteady flow for subsurface drainage problem having the parallel drains at both the side.

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APPENDIX - I

Elements of the Banded Matrix

$$W_{i,j}^{n+1,r} = \frac{w\,\Delta t}{\Delta x^2}\left[-K_I^{n+1,r} + \frac{D_{i-1,j}^{n+1,r}}{2}\left(\psi_{i,j}^{n+1,r} - \psi_{i-1,j}^{n+1,r}\right)\right] \tag{I-1}$$

$$E_{i,j}^{n+1,r} = \frac{w\,\Delta t}{\Delta x^2}\left[-K_{III}^{n+1,r} - \frac{D_{i+1,j}^{n+1,r}}{2}\left(\psi_{i+1,j}^{n+1,r} - \psi_{i,j}^{n+1,r}\right)\right] \tag{I-2}$$

$$T_{i,j}^{n+1,r} = \frac{w\,\Delta t}{\Delta z^2}\left[-K_{II}^{n+1,r} + \frac{D_{i,j}^{n+1,r}}{2}\left(\psi_{i,j}^{n+1,r} - \psi_{i,j-1}^{n+1,r} - \Delta z\right)\right] \tag{I-3}$$

$$B_{i,j}^{n+1,r} = \frac{w\,\Delta t}{\Delta z^2}\left[-K_{IV}^{n+1,r} - \frac{D_{i,j}^{n+1,r}}{2}\left(\psi_{i,j+1}^{n+1,r} - \psi_{i,j}^{n+1,r} - \Delta z\right)\right] \tag{I-4}$$

$$\begin{aligned} P_{i,j}^{n+1,r} = C_{i,j}^{n+1,r} &+ \frac{w\,\Delta t}{\Delta x^2}\left[K_{III}^{n+1,r} - \frac{D_{i,j}^{n+1,r}}{2}\left(\psi_{i+1,j}^{n+1,r} - \psi_{i,j}^{n+1,r}\right)\right. \\ &\qquad \left. + K_I^{n+1,r} + \frac{D_{i,j}^{n+1,r}}{2}\left(\psi_{i,j}^{n+1,r} - \psi_{i-1,j}^{n+1,r}\right)\right] \\ &+ \frac{w\,\Delta t}{\Delta z^2}\left[-K_{IV}^{n+1,r} - \frac{D_{i,j}^{n+1,r}}{2}\left(\psi_{i,j+1}^{n+1,r} - \psi_{i,j}^{n+1,r} - \Delta z\right)\right. \\ &\qquad \left. + K_{II}^{n+1,r} + \frac{D_{i,j}^{n+1,r}}{2}\left(\psi_{i,j}^{n+1,r} - \psi_{i,j-1}^{n+1,r} - \Delta z\right)\right] \end{aligned} \tag{I-5}$$

$$\begin{aligned} Res_{i,j}^{n+1,r} = \theta_{i,j}^{n+1,r} - \theta_{i,j}^{n} &+ \frac{w\,\Delta t}{\Delta x}\left(V_{III}^{n+1,r} - V_I^{n+1,r}\right) + \frac{(1-w)\Delta t}{\Delta x}\left(V_{III}^{n,r} - V_I^{n,r}\right) \\ &+ \frac{w\,\Delta t}{\Delta z}\left(V_{IV}^{n+1,r} - V_{II}^{n+1,r}\right) + \frac{(1-w)\Delta t}{\Delta z}\left(V_{IV}^{n,r} - V_{II}^{n,r}\right) \end{aligned} \tag{I-6}$$

APPENDIX - II

Notations

$K(\psi)$: unsaturated hydraulic conductivity (m/s);
t : time (sec);
V_x : Darcy flow velocity in the x direction;
V_z : Darcy flow velocity in the z direction;
x & z : distances along the two coordinate directions;
Δt : time stepping;
ψ : pressure head (m);
ψ_b : imposed pressure head at the ground surface;
ψ_1 : pressure head at first grid under the ground;
ψ_2 : pressure head at second grid under the ground;
θ : volumetric moisture content;
w : time weighting factor;
γ : weight coefficient;

Superscripts

n, $n+1$: refers to the values of the variables at known and unknown time levels;

Subscripts

i : refer to the grid point in x-direction;
j : refer to the grid point in z-direction;
s : refer to the side face of the domain.

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